A Physical Pendulum 1

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Introduction

A physical pendulum rotates back and forth about a fixed axis and may be of any shape. A pendulum is driven by gravity and the period depends on the strength of the gravitational field. The first pendulum all of us studied in school was a *simple pendulum:* a small mass swinging on a light string. For *small oscillations* the period of a simple pendulum is …

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad [1]$$

… independant of the mass and amplitude.

The period of a physical pendulum

Because pendulums are examples of rotational motion we shall need the moment of inertia about the centre of mass, the parallel axis theorem, torque, angular velocity, and angular acceleration.

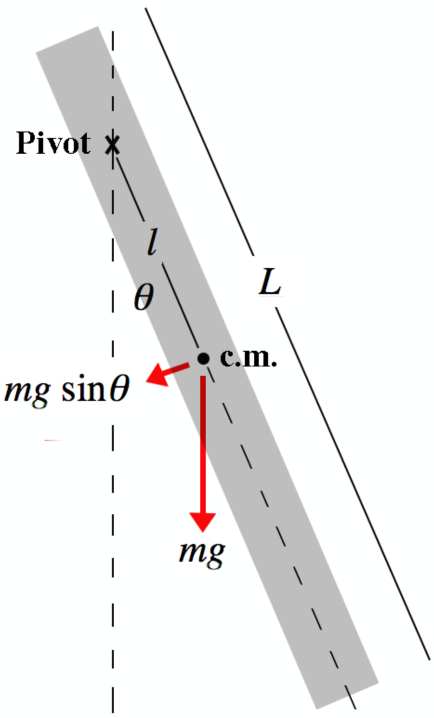


Fig 1 - A uniform thin bar if length *L* swings in an arc about a fixed point X. The weight of the bar is *mg* through the centre of mass. The component of this force that restores the bar to the vertical is -*mg* sinθ. The restoring torque is -*mgl* sinθ.

The equation of motion

Remember that …

$$T = I\alpha$$

Rearranging and writing the angular acceleration α as the second derivative of the angle theta gives …

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta \qquad [2]$$

The minus sign indicates that the torque returns the bar towards the vertical position.

For *small oscillations* for which $\sin\theta$ can be replace by the angle θ in radians, this equation describes simple harmonic motion. The solution is well known and the period for small amplitudes is …

$$T = 2\pi\sqrt{\frac{I}{mgl}} \qquad [3]$$

… where l is the distance from X to the centre of mass.

Note: For a simple pendulum the distance from the pivot to the centre of mass is *l, (*the length of the pendulum) and the moment if inertia is ml^2. Equation 3 simplifies to equation 1.

Moments of inertia

The moment of inertia of a bar of mass *m* and length *L* about a perpendicular axis through the centre of mass is …

$$I_{cm} = \frac{1}{12}mL^2 \qquad [4]$$

The moment of inertia about a pivot a distance *l* above the centre of mass is given by the parallel axis theorem as …

$$I = I_{cm} + ml^2 \qquad [5]$$

Equation 4 may be applied to a steel ruler if it is assumed to be thin. Both results are well known and proved elsewhere. They may be taken as verified in the work that follows.

Measurements with a physical pendulum

Fig 2 - A physical pendulum.

The pendulum is attached at the pivot to a Vernier angular-motion detector. The motion is plotted directly in Logger Pro.

You may wish to follow these steps

1 Find the mass and length of the bar (steel ruler).

2 Measure and record the length l from the centre of mass to each pivot point.

3 Find periods by fitting a sine function to an angle/time plot in Logger Pro (or in some other way).

4 Repeat for six to eight pivot points.

Analysis and calculations

1 Use equations 4 and 5 to calculate the moment if inertia of the steel ruler about each pivot point (assuming that the ruler is a thin uniform rod).

2 Use equation 3 to find the moments of inertia from the period measurements.

Note: calculations could be done by hand, but we recommended that data be entered into *Manual columns* and the calculations made in *New Calculated Columns* in the data logger.

See below for steps 3 and 4

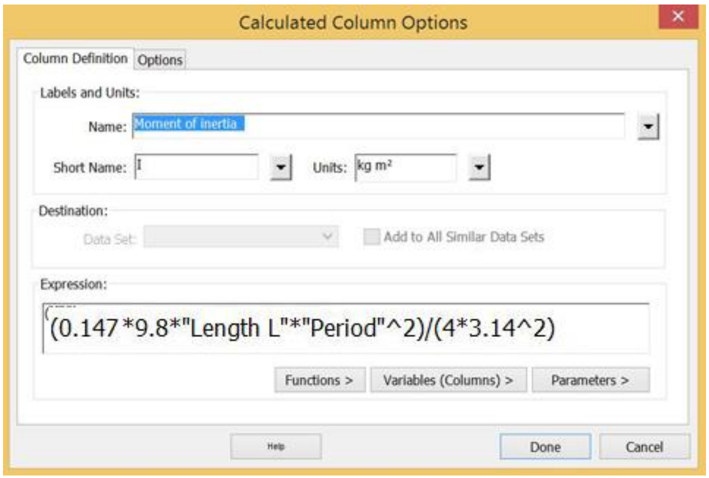
3 Compare the results of two methods by plotting the calculated set (1) against the measured set (2). Fit a straight line to the data. If both methods are reliable the line will have a slope of 1.00 and pass through the origin.

4 If the line does not (within errors) pass through the origin discuss which method may have been the most unreliable.

Notes:

A sample calculation for moment of inertia (equation 3) is shown below. The mass in this case was of 0.147 kg, Length L and Period are entered directly from existing manual columns in Logger Pro.

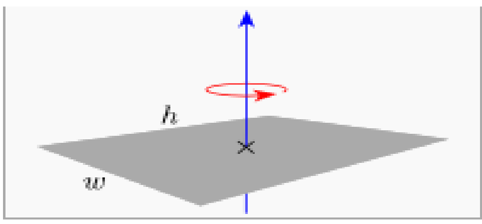
Go to *New Calculated Column* in the data menu in Logger Pro. Enter labels and the equation as shown in a *New Calculated Column*.



Calculate additional columns in the same way, being careful to write the formulae correctly.

Errors: If period measurements are repeated the value of period from the curve fit is found to vary by 2%. The lengths (L) may vary by a similar amount party because of measurement errors and partly due to irregular hole separations. Carrying these errors through a trial calculation shows that the values of moments of inertia have random experimental errors of 5%.

Extension: If your graph did not pass through the origin within errors check the accuracy of your assumption that the ruler is a thin bar. Use the moment of inertia of a flat plate about a perpendicular axis through the centre of mass (below and equation 5 to recalculate and then replot the calculated moments of inertia.



$$I_c = \frac{m}{12}\left(h^2 + w^2\right)$$