A Physical Pendulum 2

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Introduction

A physical pendulum rotates back and forth about a fixed axis and may be of any shape. All pendulums are driven by gravity and the period depends on the strength of the gravitational field. The pendulum all of us studied in school was a *simple pendulum:* a small mass swinging on a light string. For *small oscillations* the period of a simple pendulum is …

$$T = 2\pi \sqrt{\frac{l}{g}}$$

[1]

… independant of the mass and amplitude.

The period of a physical pendulum

Because pendulums are examples of rotational motion we shall need the moment of inertia about the centre of mass, the parallel axis theorem, torque, angular velocity, and angular acceleration.
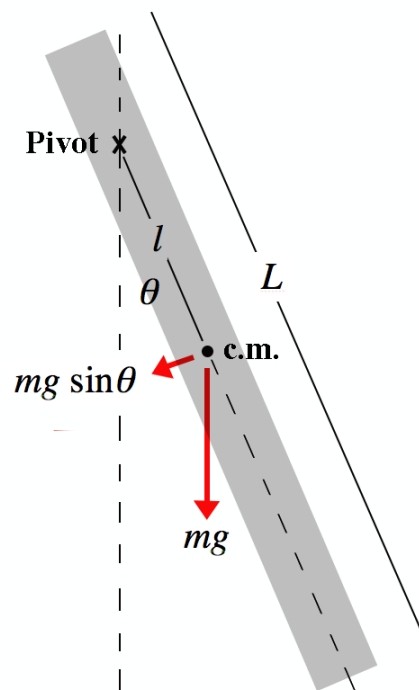
Fig 1 - A uniform bar if length *L* swings in an arc about a fixed point X. The weight of the bar is *mg* through the centre of mass. The component of this force that restores the bar to the vertical is *-mg* sinθ. The restoring torque is *-mgl* sinθ.

For angular motion …

$$T = I\alpha$$

Writing angular acceleration α as the second derivative of the angle theta gives …

$$I \frac{d^2\theta}{dt^2} = -mgl \sin\theta$$

[2]

The minus sign indicates that the torque returns the bar towards the vertical position.

For *small oscillations* for which $\sin\theta$ can be replace by the angle θ in radians, this equation describes simple harmonic motion. The solution is well known and the period for small amplitudes is …

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

[3]

… where l is the distance from X to the centre of mass.

Note: For a simple pendulum the distance from the pivot to the centre of mass is l, (the length of the pendulum) and the moment if inertia is ml^2. Equation 3 simplifies to equation 1.

Moments of inertia

The moment of inertia of a bar of mass m and length L about a perpendicular axis through the centre of mass is …

$$I_{cm} = \frac{1}{12}mL^2$$

[4]

The same relationship apples to a flat uniform rectangular plate rotating about an axis in the plane of the plate and through the centre of mass. The moment of inertia of a bar or plate about a parallel axis a distance l above the centre of mass is given by the parallel axis theorem as …

$$I = I_{cm} + ml^2$$

[5]

Both results are well known and proved elsewhere. They may be taken as verified in the work that follows.

Measurements

A rectangular wire mesh is suspended from fixed points as shown.
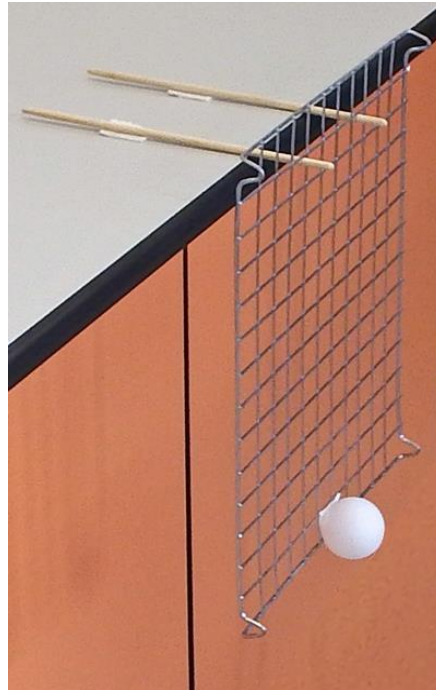
Fig 2 - A physical pendulum.

When disturbed the pendulum oscillates about an axis through the suspension points. The ball acts as an ultrasound reflector and the oscillation is plotted in Logger Pro.

You may wish to follow these steps

> **1** Find the mass and length of the mesh. **2** Measure and record the lengths *l* from the centre of mass to the axes (5 or 6). **3** Find periods by fitting sine functions with exponential damping to position/time plots in Logger Pro (or in some other way). **4** Repeat for six to eight pivot points.

Analysis and calculations

> **1** Use equations 4 and 5 to calculate the moment if inertia of the steel mesh about each axis (assuming that the mesh is a thin uniform plate).

> **2** Use equation 3 to find moments of inertia from the period measurements.

> **Note:** calculations could be done by hand, but we recommended that data be entered into *Manual columns* and the calculations made in *New Calculated Columns* in the data logger.

See below for 3 and 4

3 Compare the two methods of finding *I* by plotting the calculated set (1) against the measured set (2). Fit a straight line to the data, If both methods are reliable the line will have a slope of 1.00 and pass through the origin.

4 If the line does not (within errors) pass through the origin discuss which method may have been the most unreliable?

Notes:

A sample position/time plot
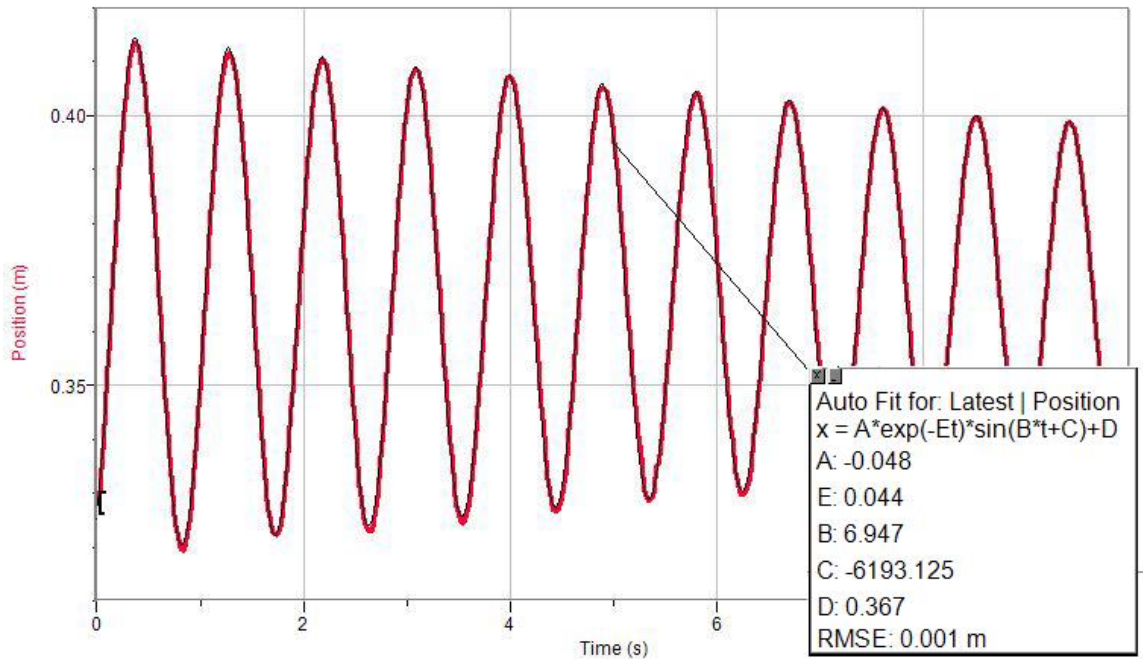
The curve fit is a *Defined Function*.

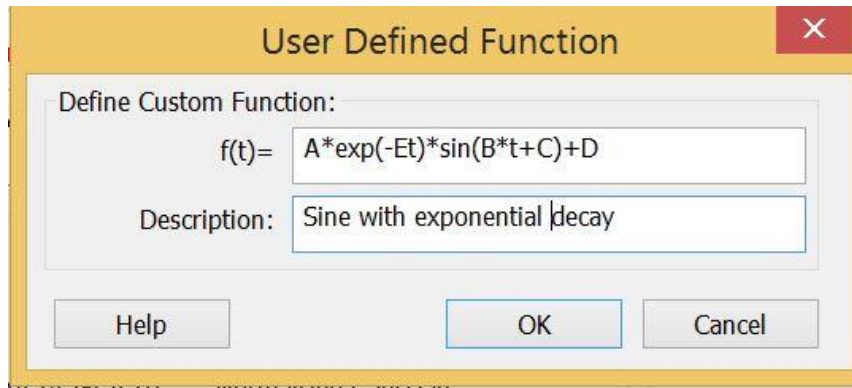**1** Go to the curve fit menu. Select sine as shown below.

2 Click *Define Function*. (See below)

User Defined Function

Define Custom Function:

f(t)= A*exp(-Et)*sin(B*t+C)+D

Description: Sine with exponential decay

| Help | | OK | Cancel |

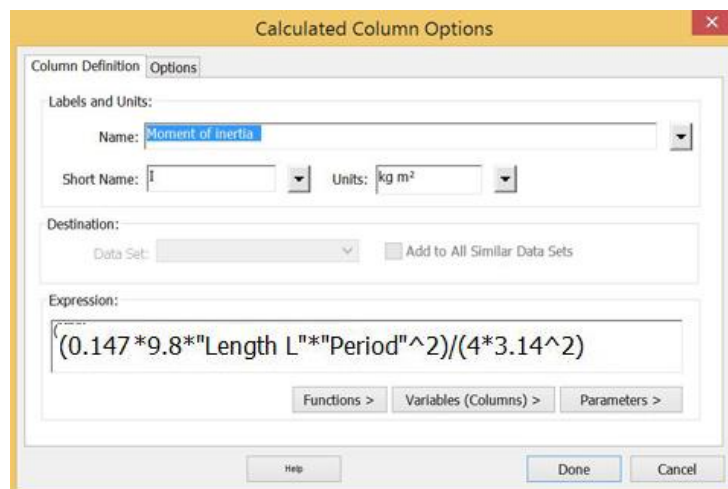Type in the exponential multiplier and click OK.

Note: In this case exponential decay is approximate because damping is due mainly to air resistance and is not proportional to velocity but to *velocity squared*. The fit is satisfactory only because there is very little damping.

A sample data table

Length L (m)	ω (rad/s)	Period (s)	I (kg m²)	I calc. (kg m²)	T²I	I²
0.15	6.78	0.926	0.0047	0.0050	0.129	0.022
0.123	6.98	0.900	0.0036	0.0039	0.100	0.015
0.103	7.08	0.887	0.0030	0.0032	0.081	0.011
0.085	7.06	0.890	0.0025	0.0027	0.067	0.007
0.0645	6.83	0.919	0.0020	0.0023	0.055	0.004
0.043	6.12	1.026	0.0017	0.0019	0.045	0.002
0.021	4.69	1.339	0.0014	0.0017	0.038	0.000

1 Data has been entered by hand in the first three *Manual columns*.

2 Go to *New Calculated Column*. To calculate the moment of inertia from the period data (column four) enter labels and the equation as shown in a *New Calculated Column*.

Calculated Column Options

Column Definition Options

Labels and Units:

Name: Moment of inertia

Short Name: I   Units: kg m²

Destination:

Data Set: ☐ Add to All Similar Data Sets

Expression:

(0.147*9.8*"Length L"*"Period"^2)/(4*3.14^2)

Functions >   Variables (Columns) >   Parameters >

Help   Done   Cancel

3 Calculate additional columns in the same way, being careful to write the formulae correctly.

Errors

In all experimental work errors are recorded with measurements, carried through calculations and results are given with errors. This process can be done in Logger pro but is the subject of a later posting.

In this case please note that if the period measurements are repeated the value of angular velocity from the curve fit is found to vary by 2%. The lengths (L) in column one may vary by a similar amount party because of measurement errors and partly due to the irregular nature of the mesh. Carrying these errors through a trial calculation shows that the values of moments of inertia have random experimental errors of 5%. Error bars have not been added to the Graph below because they are of the same size or smaller than the point protectors (triangles).

Sample graph
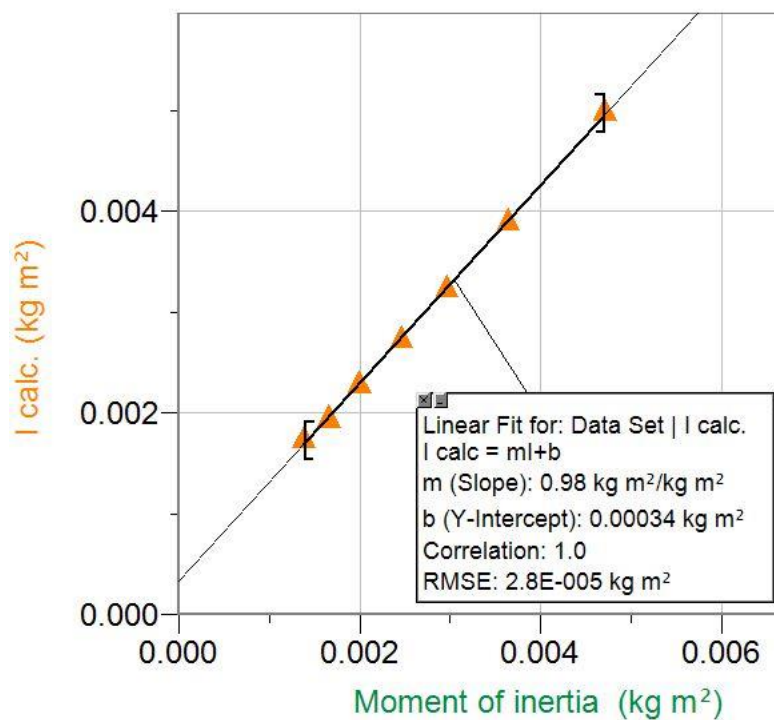
Calculated values of moment of inertia (equations 4 and 5) are plotted against moment of inertia values derived from the period measurements (equation 3) The slope is 1.0 within errors. There is a small positive intercept on the vertical axis.

Interpret the graph. *What simple correction would be required to put the graph through the origin as expected?*

When thinking about an answer to this question consider what approximations have been made in using equation 4 to calculate the moment of inertia about the centre of mass. How exactly does the mesh differ from a uniform rectangular plate?

Questions

The equation in a text describing the motion of a pendulum made from a uniform bar pivoted as shown in figure 1 is ...

$$T^2l = \frac{4\pi^2}{g}l^2 + \frac{4\pi^2}{12g}L^2$$

… where the symbols have the same meanings as those above.

a Write down equations 3,4 and 5 and derive the expression.

b Compare the equation above with the linear equation $y = mx + c$. Write down expressions for y, x, m and c.

c Sketch a graph (by hand on paper) of T^2l against l^2 using the values in the sample data table above.

d Draw an appropriate straight line on your graph and find g the acceleration due to gravity.

e Find also the length of the bar L.

f It is well known that the period of a simple pendulum (for small amplitudes) is independent of the mass. The expression above for the period of the bar does not contain the mass. Why? Is it true that the periods of all uniform physical pendulums of the same shape and size are independent of the mass?

2 A metre stick as a pendulum

a Calculate the period T for small oscillations of a physical pendulum (a meter rule) when l, the distance from the centre of mass to the axis of rotation, is 0.35 cm.

b Plot period T for this pendulum as a function of distance l from the centre of mass for values of l between 0.10 and 0.5 metres.

Hint: use either a graphing calculator or Logger pro.

c Where is the axis of rotation when the period is a minimum?

d* Can you account for this curious fact in physical terms?

3* Periods of pendulums are usually given for small amplitudes.

a Look up the period of a simple pendulum of large amplitude.

b Verify the modified formula experimentally**.**