# A physical pendulum

Water rests in a pipe of 5 cm internal diameter: see figure 2. Suspending plastic chips in salt water shows that for small amplitudes the flow is approximately laminar except for some obvious turbulence within 3 cm of each end. When displaced the slug of water oscillates about the equilibrium position with simple harmonic motion.

Since the flow is approximately laminar damping is expected to be dominated by viscous drag and proportional to the velocity. For this special case the equation of motion has an exact solution: a sine function multiplied by an exponential decay factor.

The water oscillation was videoed and displacement plotted point by point in Logger Pro.

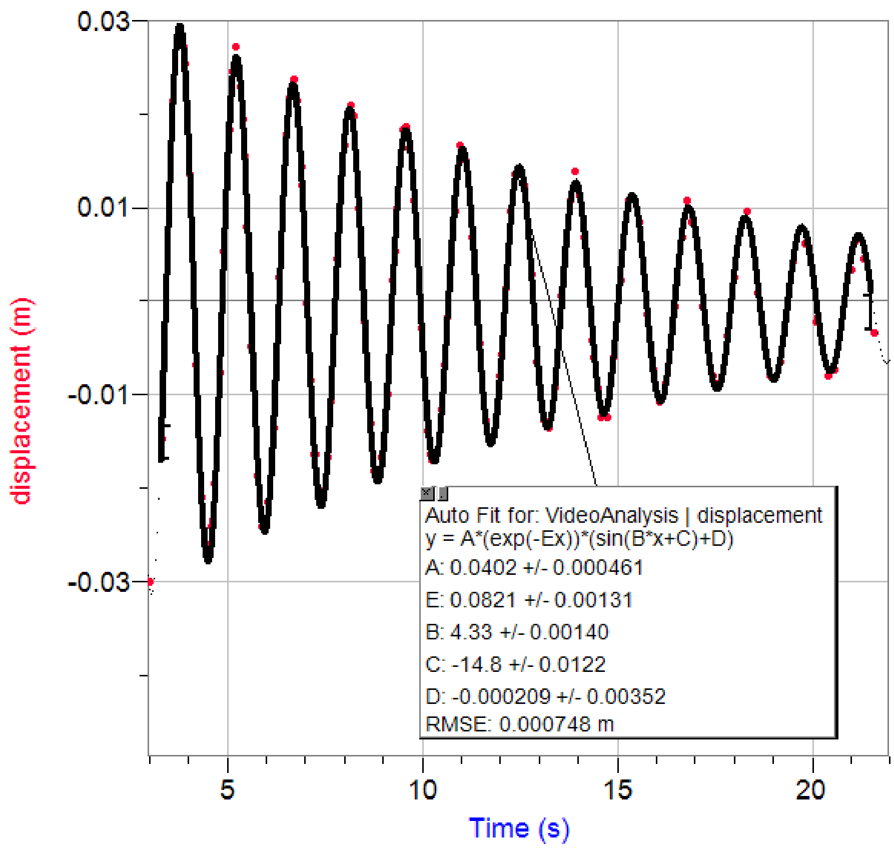


**Fig 1** - auto-fit of a sine function with exponential amplitude decay.

The angular frequency is 4.33 rad/s. The measured period of the motion, which is independent of the amplitude, is 1.45 s. The period can be calculated by considering the water to be a physical pendulum oscillating about the centre of curvature.

The period relationship for a physical pendulum is …

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$ ….. [1]

… where and $I$ is the moment of inertia about the pivot ($mr^2$)
and $l$ is the distance of the centre of mass from the centre of rotation.

The centre of mass of the circular water arc is marked on the centre line below.

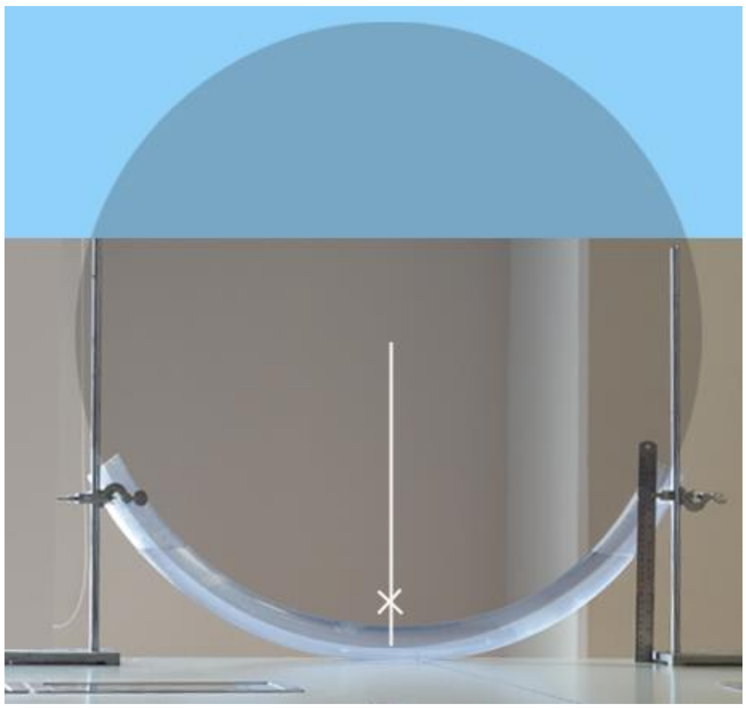

**Fig 2** – the circular arc of water in the 5 cm diameter pipe.

The diameter of the circular arc was 0.46 m to two significant figures. The position of the centre of mass of the water (marked above) can be shown to be 0.90$r$ from the centre of curvature. The period is given by equation 1 as 1.44 s. The measured period from the curve fit (figure 1) to three significant figures, is 1.45 s, which is in excellent agreement within errors.

*The reader may show for themselves, by taking measurements on the figure and integrating, that the centre of mass of the uniform circular arc in this case is 0.90r from the centre of curvature.*

## Appendix I

### Reynolds number

Reynolds number is widely used to predict the onset of turbulence when objects fall in air or water and when air or water flows in pipes. The number is calculated as a velocity by a length (diameter of a pipe) over the viscosity of the fluid. It is found that if Reynolds number is below ~ 2300 steady one directional flow in a pipe is laminar and if $R_e$ is greater than ~ 4000 the flow is turbulent. What happens in the transition region depends on the details of the situation.

$$R_e = vD/\nu$$

… where v is the flow velocity, $D$ is the diameter and $\nu$, (nu), is the kinematic viscosity of water ~ $0.90 \times 10^{-6}$ Pa. s at 25 °C. For an amplitude of 3 cm the maximum velocity of water in the tube ($A\omega$) is 0.013 m/s.

$$R_e = 0.013 \text{x} 0.05/(0.9\times10^{-6})$$

$$= 700$$

… showing that unidirectional water flow in the pipe at this velocity would be laminar.

Direct observation shows that for this oscillation, turbulence is confined to the ends of the water column, and that the flow over 95% of its length is laminar or nearly so. If the pipe diameter is reduced from 5 to 2 cm the flow becomes gently turbulent over the whole length of the pipe. Drag on the walls is a more important factor in generating turbulence in the oscillating water in the narrower pipe. Reynolds number is a guide, but in this situation direct observation is required.

### Additional comments

Reynolds number was proposed by Stokes in 1851 and was popularized by Reynold in the 1880’s. Reynold was a professor of Engineering (appointed at 26) and was an excellent mathematician.

Most common flows involve at least some turbulence. Dry leaves and raindrops fall in air in turbulent flow. When tea is poured from a pot and coffee is stirred the flows are turbulent.

Laminar flow and the transition to turbulence can be seen as smoke rises in still air from a joss (incense) stick. Water droplets in clouds fall in laminar flow when less than 100 microns in diameter. Clay particles in water settle slowly in laminar flow, with a sedimentation rate that is related to the diameter of the particles.

**Appendix II**

**The centre of mass of the circular arc of water**

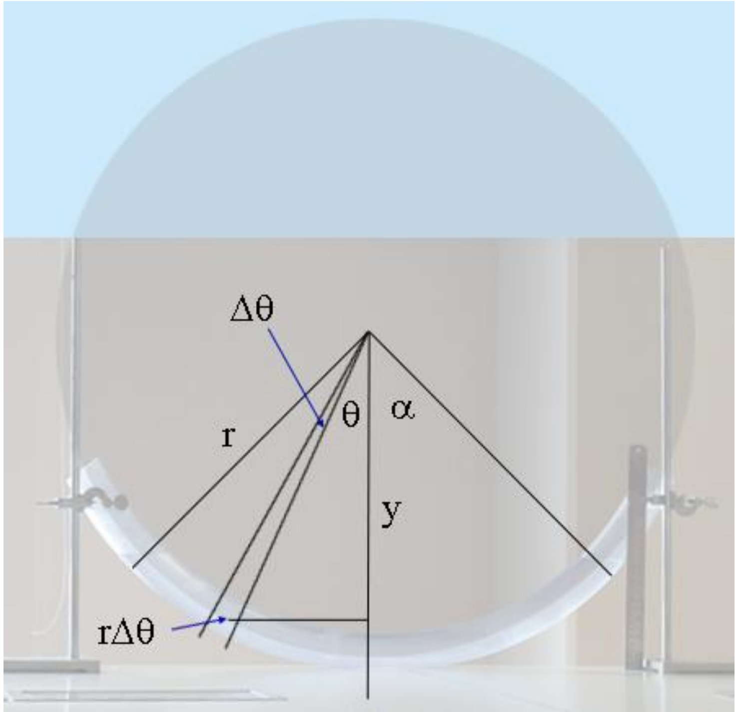


**Fig 3** – the water arc length L is 2rα.

Figure 3 defines the arc length as $2r\alpha$ and an element of arc length as $r\Delta\theta$. Since the arc is uniform, its length can be taken as representing the total mass, and $r\Delta\theta$ as an element of mass $\Delta m$. The centre of mass will lie on the y axis a distance $\bar{y}$ from the centre of curvature. Looking carefully at the diagram (figure 3) the value of $\bar{y}$ is given by ….

$$\bar{\mathbf{y}} = \frac{\int_{-\alpha}^{\alpha} r^2 \cos\theta \, d\theta}{r \int_{-\alpha}^{\alpha} d\theta}$$

Integrating between the limits - α and α and cancelling $r$ gives the vertical distance to the centre of mass as …

$$\bar{\mathbf{y}} = \frac{r \sin\alpha}{\alpha}$$

$= (0.702/0.778)\, r$ … when α is 44.6°

$= 0.90\, r$