Air pressure as the rate of change of momentum

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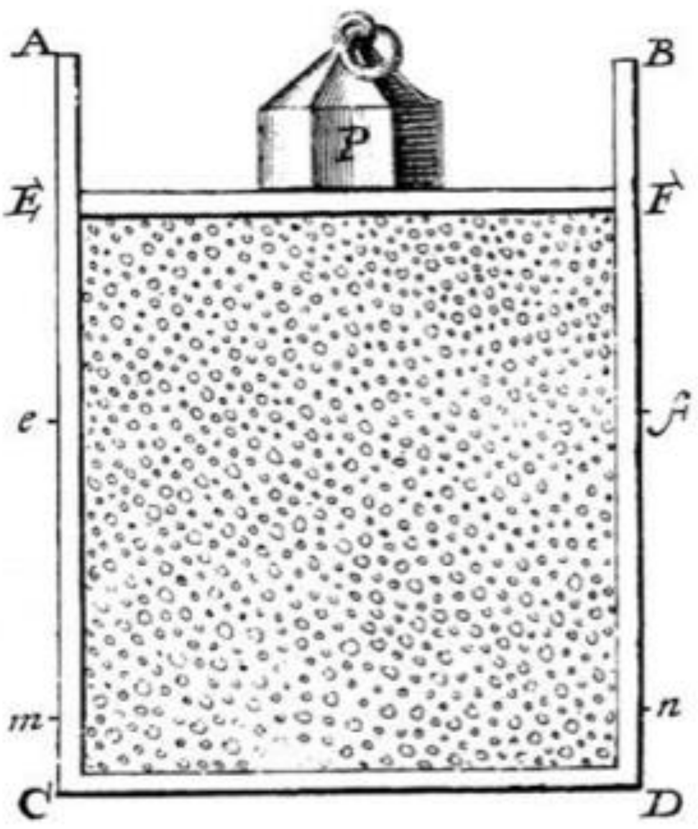
Introduction

In 1738, nearly three hundred ago, Daniel Bernoulli understood that gas pressure could be explained as the bombardment of the walls of a container by fast-moving, elastic particles. His hand-drawn diagram in the style of that time shows a weight supported in this way.



Even scientists at that time ignored Bernoulli. For a hundred years they just went on believing that a gas was hard to compress because of repulsion between molecules. Newton (in England) had shown that an inverse-square force law gave the correct PV relationship, which matched what people believed. In 1820, another attempt to put forward a kinetic theory was made by John Herepath in England. His paper was rejected by Sir Humphrey Davy (then president of the Royal Society) on the grounds that to associate temperature with the energy of fast moving particles implied the existence of an absolute zero of temperature: an idea that he would not accept.

It was not until 1859 that Maxwell tackled the question, derived the velocity distribution, and finally ended the belief in repulsive forces.

Compressed air

A 2.5 ml syringe is closed with a plug of clay (blue in figure 1). The cross sectional area of the barrel is 0.5 cm^2. The weight is 55 N.

Fig 1 - 55 newtons rest on a syringe. The weight is supported by the compressed air in the small space above the blue clay plug.

The downward force on the piston is 55 N, plus the force due to external air pressure 5 N. If gas pressure is due to the impacts of fast moving molecules on the face of the piston the momentum change per second of these molecules the must equal 60 newtons. That is difficult to believe: the total mass of 2.5 ml of air is 0.003 grams. Molecules would have to be moving very fast to do that.

Explanation

Imagine the air replaced by three particles moving in perpendicular directions (figure 2). Only the vertical motion of one third of the mas of the trapped air (0.001) grams contributes to the momentum change on the piston.

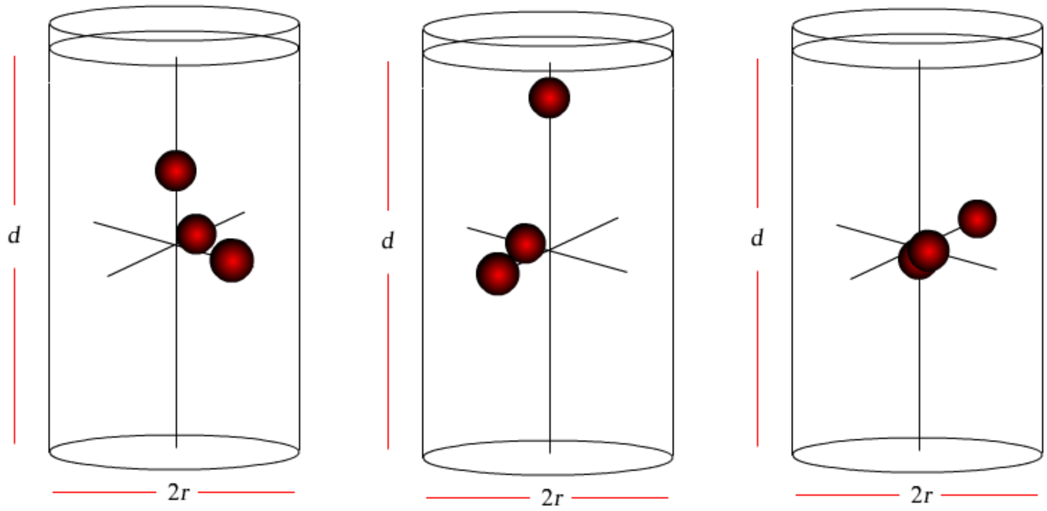


Fig 2 – three balls moving with average molecular velocity *v* replace the air in the syringe.

When the weight has pushed the piston down to a distance *d* above the clay plug, the distance between impacts of the one ball moving in the vertical direction with velocity *v* is 2*d* and time between impacts Δ*t* is …

$$\Delta t = 2d/v.$$

The momentum change is 2*mv* times *N (the number of impacts per second*).

$$\Delta mv \ = 2mvN$$

The momentum change per second is …

$$\Delta mv/\Delta t = f = 2mvN \text{ x } v/2d \ = mv^2/d$$

In the real case shown in figure 1 the distance between the piston and the bottom of the syringe is $3.0 \text{x} 10^{-3}$ m. The mass of the imagined particle is $1.0\text{x}10^{--6}$ kg and the force on the piston is 60 N.

$$v^2 = fd/m = 60\text{x}3\text{x}10^{-3}/1\text{x}10^{-6} = 180\text{x}10^3$$

$$v \ = 420 \text{ m/s}$$

The calculation gives the average velocity of air molecules (mostly nitrogen) in the syringe. The value is approximate because the values used to calculate it are approximate, and no allowance has been made for friction between the wall and the piston. *Molecules are fast.* The average speed of air molecules at STP is about 500 m/s: faster than the speed of sound, which is 340 m/s.

A second example

a The 1.28 kg of air (mostly nitrogen) contained in a cube one metre on a side is replaced by three elastic balls of mass 0.426 kg oscillating at right angles to the faces.

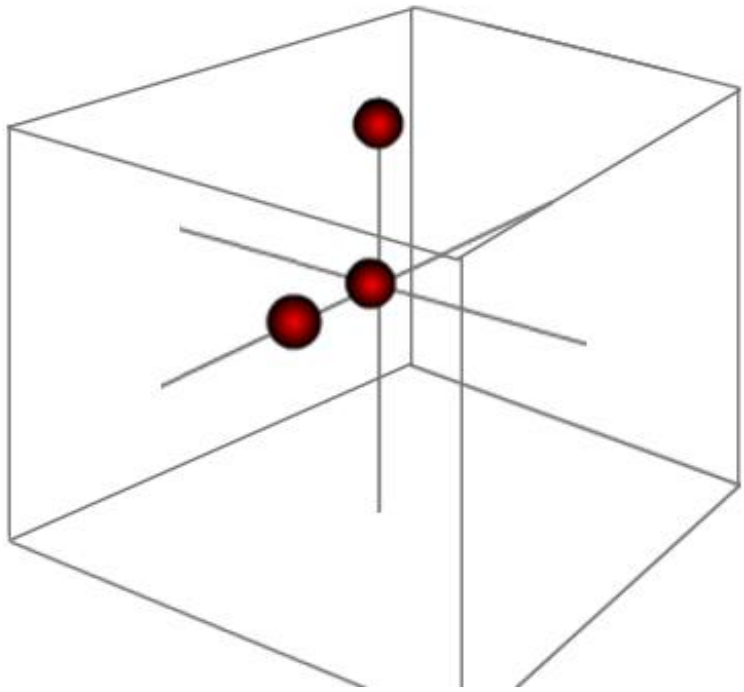


Fig 3 – three balls moving with average molecular velocity *v* replace the air in the cube.

The force on the top face *f* is 100,000 N because air pressure is 10^5 Pa.

The distance between faces *d* is 1.00 m.

$$v^2 = fd/m = 1\text{x}10^5/0.426$$

$$v = 470 \text{ m/s}$$

The average velocity of air molecules at STP is 470 m/s, a more accurate value than the estimate above.

b By replacing the air in the cube with an appropriate number of different masses find the average velocities of N_2 and O_2 molecules in air at STP.