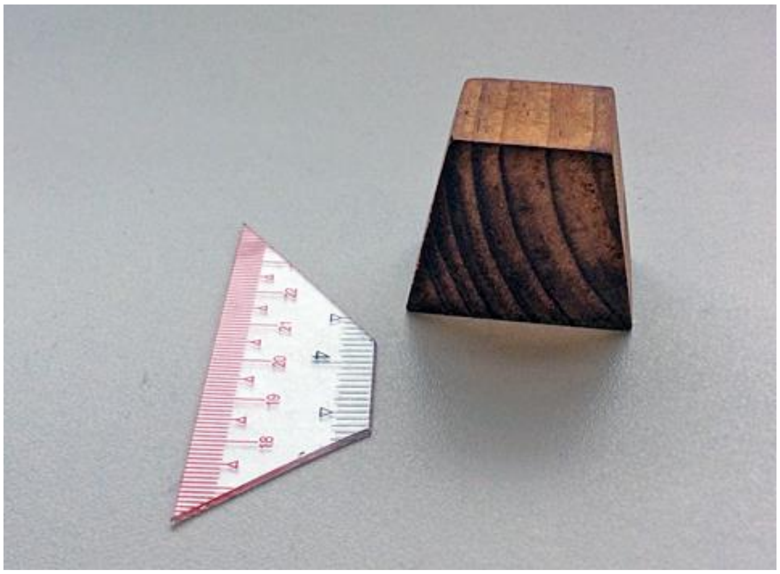
Area, volume and density, with measured values

1. You have a mm scale and a block of wood. Please read the first page of *Error Propagation* [pdf] in *physics@KVIS: Readings in Physics*.



You are asked to find the area of the plastic ruler and the density of the wood using length measurements with errors, given that the mass of the block is 14.84 ± 0.01 g.

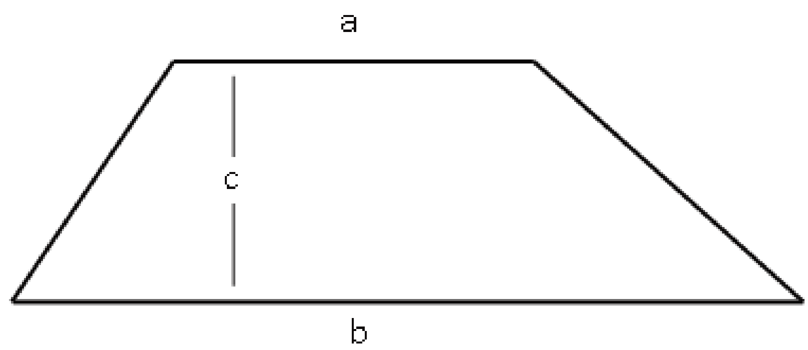
i *How accurately can you read the mm scale?*

ii *How regular is the block? Within what limits are the opposite faces parallel?*

iii *How will errors in final values be found quickly and efficiently?*

Solutions

1 Trace the outline of the plastic on paper with a sharp pencil and measure the lengths on the diagram. Errors in length measurements will be (by convention) half the smallest scale division (± 0.5) mm: or, in more detail, the combination of many smaller errors, including uncertainties in copying the shape, small errors in the scale itself and errors in estimating the last decimal place at both ends of the scale when measuring. These errors add to approximately ± 0.5 mm.



The lengths are … $a = 2.60 \pm 0.05$ cm
$b = 7.20 \pm 0.05$ cm
$c = 2.90 \pm 0.05$ cm

The area is ½(a+b)c = 0.5 (2.60+7.20) 2.90 = 14.2 cm^2

You must now calculate the likely error to one significant figure. To find the upper bound (how large the error could be) add the errors in the sum (a + b) and add the fractional errors (percentage errors) in the product.

$$\tfrac{1}{2}(a+b)c = 0.5\,\mathbf{[}(2.60 \pm 0.05) + (7.20 \pm 0.05)\mathbf{]}\,(2.90 \pm 0.05)$$

$$= 0.5\,(9.80 \pm 0.1)\,(2.90 \pm 0.05)$$

… adding the errors in a and b.

The fractional error relationship is ….

$$\Delta xy = XY(\Delta x/X + \Delta y/Y) \quad \text{… where } X = (a+b) \text{ and } Y = c$$

$$= 28.4[(0.1/9.8) + (0.05/2.9)]$$

$$= 0.8$$

$$\text{Area} = \tfrac{1}{2}(28.4 \pm 0.8) = 14.2 \pm 0.4 \text{ cm}^2$$

Note:

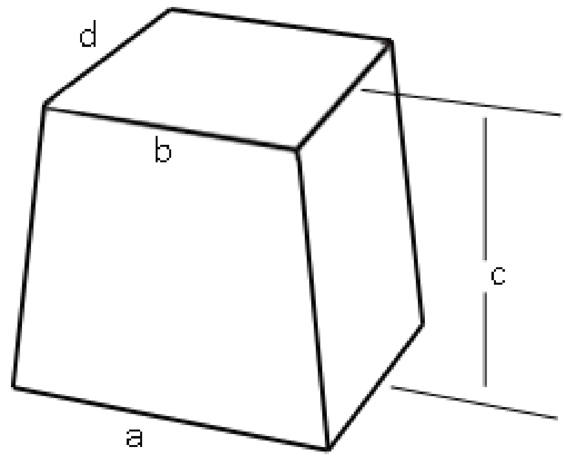
Percentage errors can often be added by inspection.

$$1.0\,\% + 1.7\,\% = 2.7\,\%$$

… and 2.7 % of 14.2 is 0.4 to one significant figure.

Many short questions in examinations are designed to test that you understand the principles and can be done quickly by adding percentage errors.

2 The dimensions of the block are defined on the diagram.



Mass = 14.84 ± 0.01 g

Errors in length measurements are, by convention, ± 0.5 mm: or, in more detail, a combination of small errors in estimating the last decimal place on the scale and small differences in the lengths of the sides of the block, which is almost a right trapezoidal prism with parallel end faces.

The lengths are …

$a = 3.85 \pm 0.05$ cm
$b = 2.40 \pm 0.05$ cm
$c = 2.90 \pm 0.05$ cm
$d = 2.50 \pm 0.05$ cm

The area of the front face … $\frac{1}{2}(a+b)c = 9.1 \pm 0.3\ \text{cm}^2$.

The volume of the block … $\frac{1}{2}(a+b)cd = (9.1 \pm 0.3)(2.50 \pm 0.05)$

$= 23 \pm 1\ \text{cm}^3$

The density ... $\rho = (14.84 \pm 0.01)/(23 \pm 1)$

$= 0.65 \pm 0.03\ \text{g/cm}^3$

…where the errors are calculated as upper bounds.

The reader should check the error calculations, by first adding percentage errors by inspection, and then by finding the fractional errors in the products.

Post script

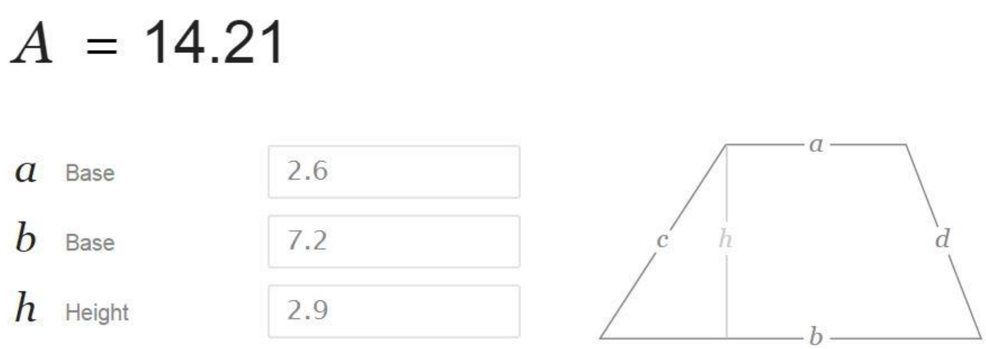
The methods above (adding absolute errors in sums and adding fractional or percentage errors in products and quotients) gives reliable upper bounds to errors carried through calculations. When other functions (sines and tangents) are involved, or a quick reliable calculation is needed, an error can always be found by calculating twice: once without errors and a second time, making the result as different as possible using the stated errors. The error is then written down to one significant figure by inspection.

For example: repeating solution 1

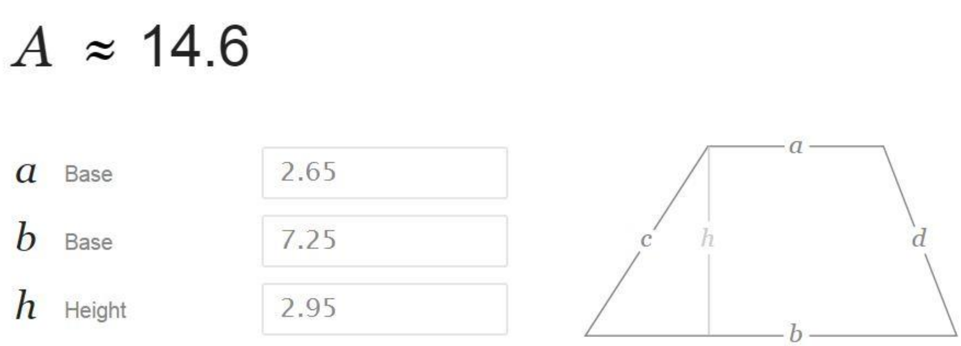
We use the calculator at …

https://www.google.co.th/?gws_rd=cr&ei=1LgjWYFMoPo0ASFkbaYDA#safe=off&q=area+of+a+trapezium

Calculate the area …



Follow with a second calculation …



The result with error is (as above) ….

$$14.2 \pm 0.4 \text{ cm}^2$$