Black Body Radiation

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Electromagnetic radiation

Energy absorbed as heat on the surface of the earth is transferred from the sun by electromagnetic radiation, (with no transport by convection or conduction). The earth receives Xray, UV, visible and infrared solar radiation with the peak intensity in the visible region. Short wavelength ionizing radiation (Xrays and far UV) heats the stratosphere and doesn't reach ground level but the atmosphere is transparent to the near UV (that causes sunburn), to visible light, and to near IR wavelengths. Before we can discuss the energy balance between solar input, reflection and re-radiation from the earth and begin to think about global warming we need to discuss *black body radiation*.

What exactly is a black body?

I can guess what you're thinking, and that is (as you will see) not right.

A black body is one that absorbs all incident radiation of all wavelengths. Because it absorbs all visible light it will look back to your eyes but that is only one small part of it. A truly back body doesn't exist in nature and cannot be made. The closest thing you will find at your house is the hole in a clay ball after the gunpowder has been burnt.

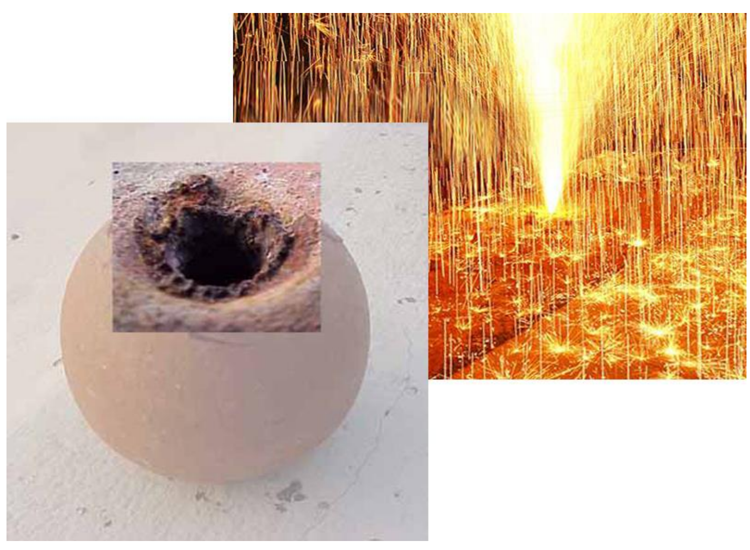

Fig 1 – a nearly black body.

The ball is coated on the inside with soot (carbon black). Radiation entering the hole (including the light you can see) is mostly absorbed. Very little of what is reflected comes back out the hole. When you look in the hole you are looking at an almost true black.

Black body radiation

If you heat the empty clay ball to 700 K (400°C) and look in the hole in darkness you will see a faint red glow. The ball has become what is known as a *cavity radiator*. What you are seeing is just the small visible part of a radiation spectrum that extends all the way into the far infrared. The spectrum of this almost black body is shown in figure 2.

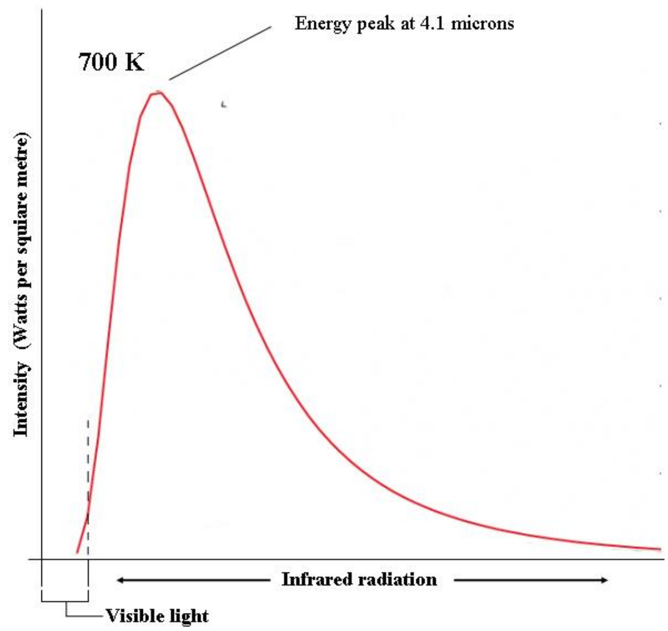

Fig 2 – the black body radiation curve for 700 K. Emission intensity in W/m^2 is plotted against wavelength.

Note the cut-off frequency at short wavelengths. The cavity is emitting *no* radiation in the green and the blue. Maximum intensity is in the near infrared and the upper tail of the distribution extends to longer wavelengths without limit.

There is a mathematical equation for this curve in two variables, temperature and wavelength that was first written down by Max Planck around 1900. It is important in the history of physics because it provided the first example of quantum mechanics. You could look for it on the web but for now we need to know only the integral (the area under the curve) and the peak wavelength that are both simple easily-remembered relationships.

At different temperatures the curve has the same shape. At higher temperatures the intensity rises and the peak and the short wavelength cut-off move to the left. For a black body at the temperature of the surface of the sun (6000 K) the cutoff is in the UV and the peak is in the green (the middle of the visible spectrum). See figure 4 below.

At lower temperatures the intensity reduces and the peak shifts to longer wavelengths. Your own body with a skin temperature of 35°C (312 K) is almost a black body radiator. That is true. Everyone is the same. In the infrared white and dark skin are both *black bodies*.

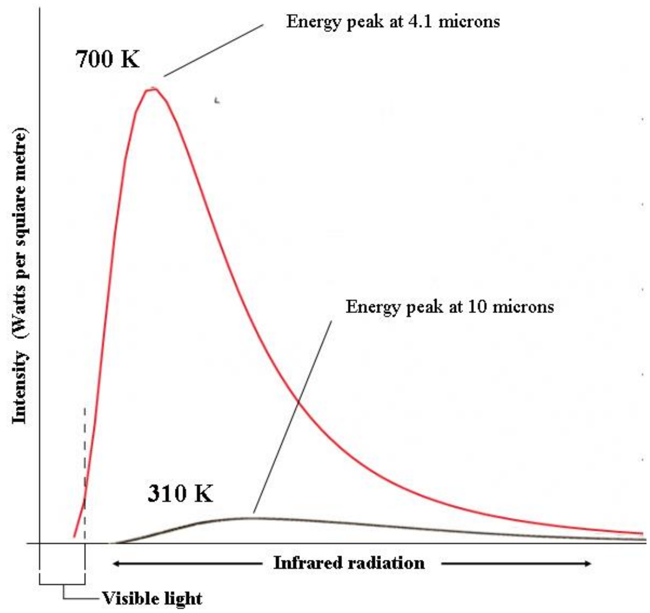

Fig 3 – a black body radiation curve for 310 K (the human body) on approximately the same vertical scale.

At 35°C the power radiated in W/m^2 is much less, the short wavelength cut-off has moved into the infrared, and the peak intensity is now at a wavelength of 10 microns.

Stefan's law

The heat radiated per second $\Delta Q/\Delta t$ in W/m^2 radiated over all wavelengths from a black body of surface area A at temperature T (kelvins) is the area under the black body curve, which is given by …..

$\Delta Q/\Delta t = \sigma A T^4$ … where sigma is the Stefan-Boltzmann constant $5.67\times10^{-8}\ Wm^{-2}K^{-4}$.

The law was derived by Josef Stefan in the 1879 from experimental data. Boltzman (Stefan's most famous student) refined his law to include bodies that are more gray than black by inserting a second constant ε defined as emissivity. In particular the emissivity of human shin is about 0.5 in the visible region and rises to 0.98 in the infrared. *Human skin (of any colour) is an almost perfect black body radiator at normal skin temperatures.*

Values of ε at infrared wavelengths listed on the web range from near zero to one.
http://www-eng.lbl.gov/~dw/projects/DW4229_LHC_detector_analysis/calculations/emissivity2.pdf

Wein's law

The peak wavelength of the black body curve *in metres* is related to the absolute temperature by …

$$\lambda_{max} = 0.0029/T$$

These two relationships, Stephan's law and Wein's law, are sufficient for now. If Plank's relationship for the black body curve is required it may be found on the web.

The emissivity of the sun

The sun is a common type of star of average mass and lifetime, classified by astronomers as a G2V star. The apparent surface temperature is 5777 K but it is not strictly a black body radiator as can be seen on the graph below.

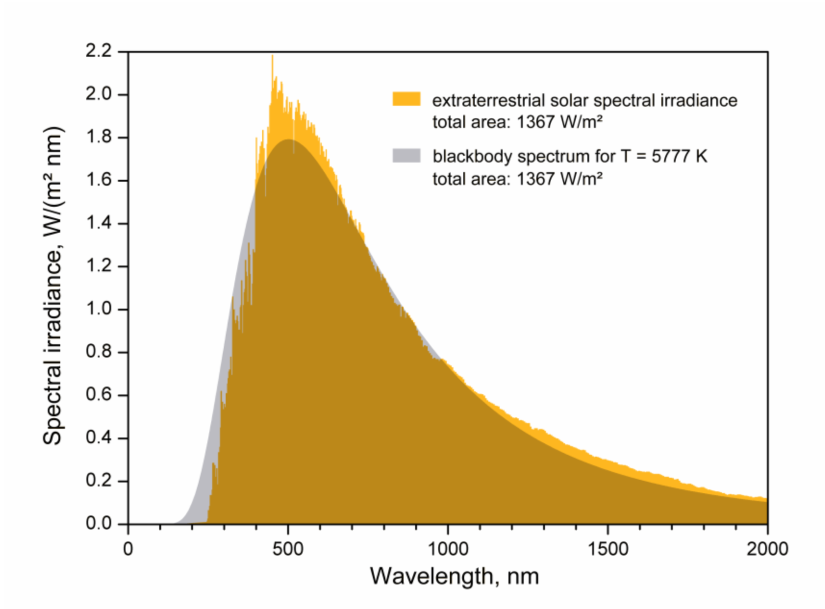

Fig 4 – a black body radiation curve for 5777 K compared to the solar output.
https://commons.wikimedia.org/wiki/File:EffectiveTemperature_300dpi_e.png

The solar spectrum is not a perfect match to a back body curve but selecting the best fit temperature gives an emissivity that is close to 1.00.

The intensity of solar radiation above the atmosphere is 1360 W/m^2. The radius of earth orbit is 1.496x10^{11} m and Stefan's constant is 5.67x10^{-8} Wm^{-2}K^{-4}. *Repeat Stefan's calculation of the surface temperature of the sun with modern values.*

Notes: Stefan's estimate with the values he had at the time was 5703 K. This was the first time the surface temperature had been found to this accuracy. *This biography of Stefan is worth reading if you have time.*

http://www-history.mcs.st-and.ac.uk/Biographies/Stefan_Josef.html