Centre of mass

Demonstrations and/or activities

1 The distance of the centre of mass $\bar{x}$ from an arbitrary origin for a line of point masses if found by calculating the sum …. $m_1x_1 + m_2x_2 + m_3x_3 +$ ..... and dividing that by the total mass … $m_1 + m_2 + m_3 +$ …. . where x_i is the distance from the origin to m_i.

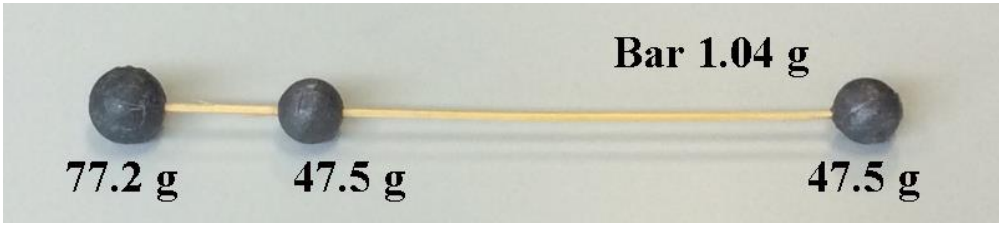

Fig 1 – lead balls.

To reduce calculation the centre of the left hand mass is selected as the origin, the balls are taken as being small and the mass of the bar is neglected. The calculation locates the centre of mass on the bar a little to the right of the central mass. The position is confirmed by demonstration.

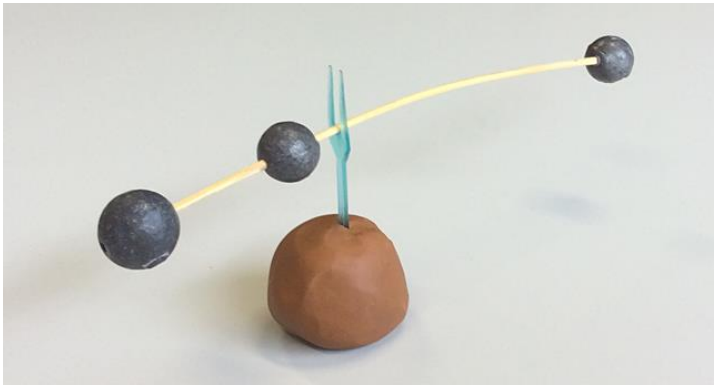

Fig 2 – balancing confirms the location of the centre of mass.

Note: the centre of mass in figure 2 is below the point of support because the skewer has bent downwards.

At a later time the centre of mass can be calculated again with spherical masses and without neglecting the mass of the bar. Little difference will be found.

2 The centre of mass of a two dimensional array of point masses is found in the same way by finding the coordinates separately.

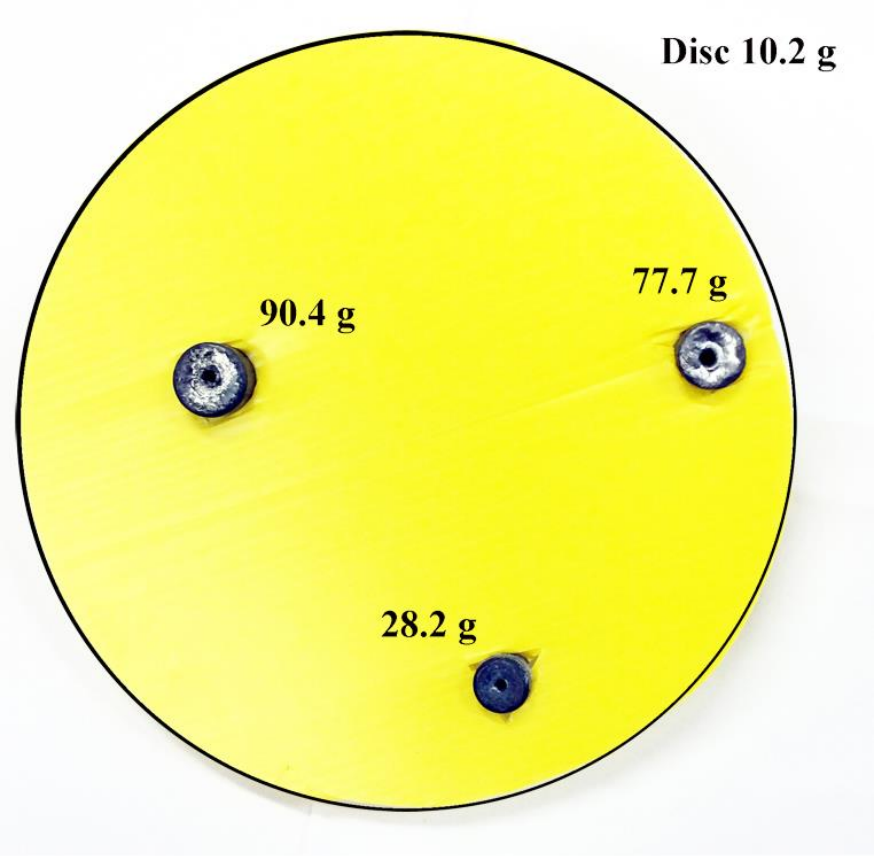

Fig 3 – lead embedded in a plastic disc.

Axes are selected to pass through as many centres as possible to reduce calculation. The coordinates of the centre of mass ($\bar{x}$, $\bar{y}$) are found one at a time
by measuring and calculating as above, neglecting the mass of the disc in the first instance.

> At a later time the centre of mass can be calculated again with cylindrical masses and without neglecting the mass of the disc. Little difference will be found.

2a The moment of inertia about the centre of mass as the sum …

$$m_1v_1^2 + m_vv_2^2 + m_3v_3^3 + ...$$

Verify the calculated value with an angular motion detector and a known torque.

3 Provide the equipment in figure 4. Ask for the mass of the coin if the 'weight' of the paper is 90 g/m^2.

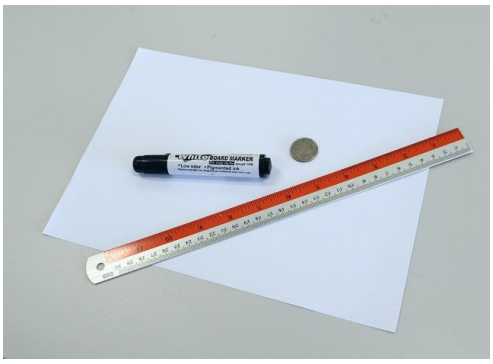

Fig 4 – equipment.

As a demonstration or activity, crease the paper to make a bar. Place the coin on one end and balance the bar on the pen. *The weight of the paper acts through the centre of mass* and the result follows. When balanced, the centre of mass of the paper and coin taken together is on a vertical line through the pivot.

4 Wine bottle display-stands are cleverly designed to provoke interest.

Fig 5 – wine bottle display-stands.

Ask the assembled to locate the centres of mass, and to discuss the stability in each case.

5 When standing on one foot the centre of mass of the figure must be on a vertical line that passes inside the foot (the loop of wire on the table).

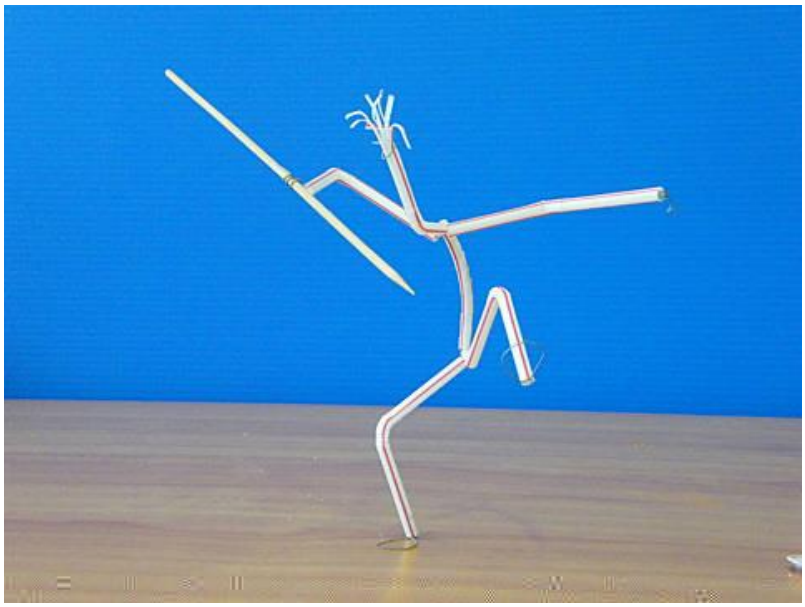

Fig 6 – standing figure with spear.

The model in figure 2 is made with wire and bendy plastic straws. The spear raises the centre of mass. If the spear is overhead it becomes more difficult to adjust for standing on one foot.

Notes on construction

Use thin wire so the figure flexes just a little when standing. It is important to make the loops that form the feet flat, and to make them as shown, not too small, but not so large as to look ungainly. (*Remember: taller people have longer feet*).

The proportions are approximately human but the legs are in equal sections and the arm and leg straws are the same lengths.

Make the feet by bending around the thumb and add the leg straws. Twist the leg wires to make the back (adding a third shorter wire to form the neck and head-support). Cut the back straw half way through every few mm and then thread it over the raised wires that will form the arms. Add the head and the arm straws. Trim and twist the ends of the arm wires to make hands.

Standing up: press one foot to a flat bench with one hand and adjust the lean of the ankle in stages with the other until you have it right by trial and error. Very instructive ….