Conceptual questions

Centre of mass

Recall the relationships …

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots} \qquad \bar{v} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots}{m_1 + m_2 + m_3 + \dots} \qquad \bar{a} = \frac{m_1 a_1 + m_2 a_2 + m_3 a_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

…. where the symbols have their usual meanings.

1 Suppose a plane in level flight drops a continuous stream of identical bombs one at a time at equal intervals Δt.

> **a** Neglecting air resistance and using the expression above show that the vertical acceleration of the centre of mass of the collection of falling bombs is g. Show also that the result holds if the bombs are not of equal mass.

> **b** A student argues that since a collection of masses can be replaced by a single mass at the centre of mass, the velocity of the centre of mass of the falling bombs will be gt.

> > **ii** Why is the student not correct?

> > **ii** Using the expression above for the velocity of the centre of mass, show that the correct expression is $gt/2$ where t is the total time for which the stream of bombs has been falling.

2 Three lead balls with masses of 30, 60 and 90 g are embedded in a uniform polystyrene disc of 10 g.
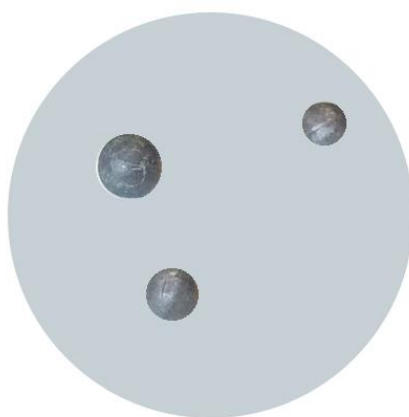
> **a** Is a student justified in neglecting the mass of the disc in a calculation of the centre of mass with measured distances?

> **b** How could axes and an origin be defined to reduce the measurements and calculation required as much as possible.

3 Two asteroids A and B with a mass ratio of 2:1 are in mutual orbit.

      a Neglecting the influence of more distant bodies, briefly describe the motion of the centre of mass.

      b Sketch likely orbits in the frame of reference of the centre of mass.

      c Sketch these orbits in the frame of reference of a passing astronaut.

   **d** Sketch these orbits in the frame of reference of the larger asteroid.

4 Pairs of asteroids in mutual orbit are known and planets have collections of small moons. Some of these moons are believed to have been captured from the asteroid belt.

      a Small asteroids are known to quite often pass within the orbit of the moon, and very occasionally to impact the earth. Given the frequency of these encounters why is moon capture a rare event?

      b Explain briefly why a gravity-assist maneuver (when a spacecraft has kinetic energy increased in a close flyby of a planet or moon) is not possible in a two-body interaction. You may like to read this article
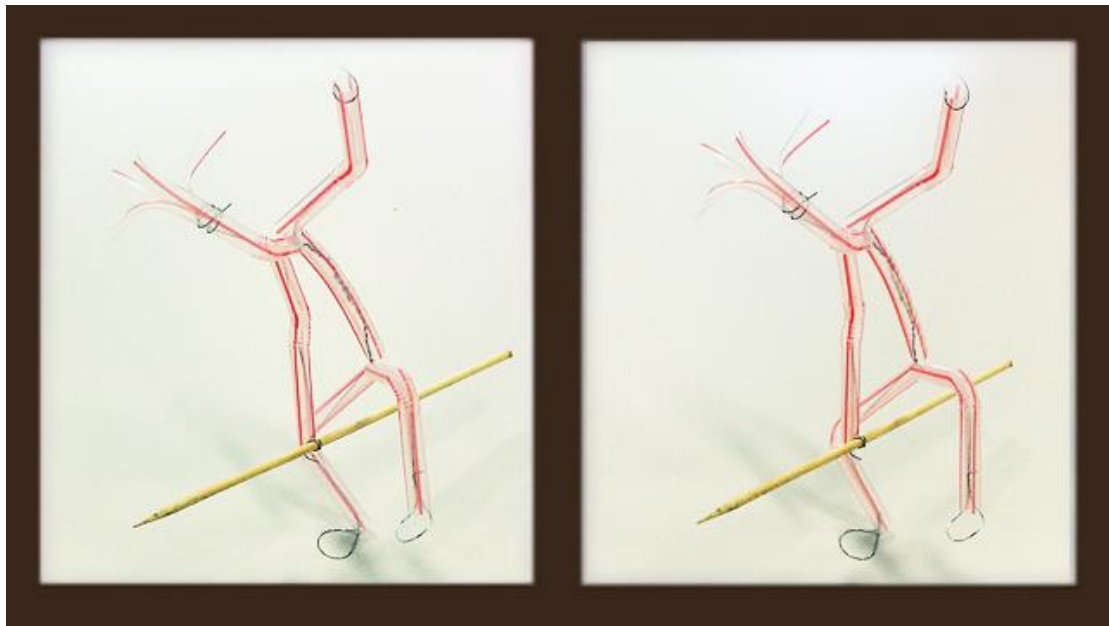https://en.wikipedia.org/wiki/Gravity_assist#Explanation

5 Discuss the physics that you see in the illustration below.
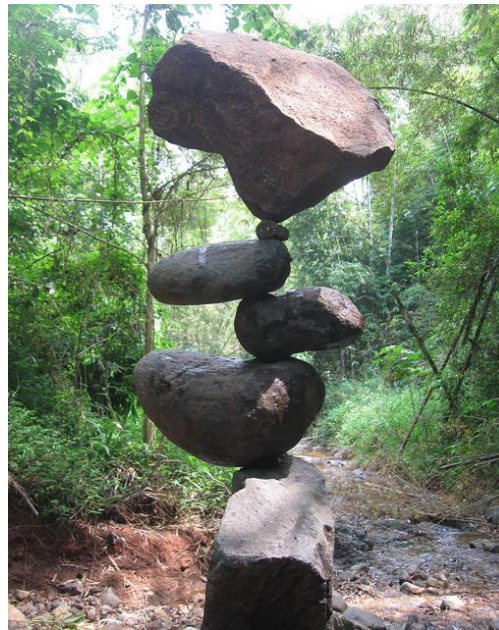
https://en.wikipedia.org/wiki/Rock_balancing#/media/File:Rock_balancing_(Counter_Balance).jpg

6 The Standing figure is made from wire and drinking straws.

The illustration is a cross-view stereo pair. If you cannot cross view in 3D just use the single images.

    a Locate the centre of mass by inspection as closely as you can.

    b What advice would you give about the size and shape of the wire loop that makes the foot to a someone making this model for the first time?

    c A student finds it to be much more difficult to adjust the figure to stand on one foot when the spear is raised above the shoulder. Explain why.