# Coefficients of Restitution

The coefficient of restitution for a two-body collision is defined as the ratio of relative velocity of retreat over relative velocity of approach. When a ball bounces vertically on a level floor attached to the earth the ratio simplifies to the rebound velocity over the impact velocity. Since the kinetic energy of the rebound $^1/_2mv^2$ is converted to potential energy $mgh$ as a body rises, the coefficient of restitution is the square root of the ratio of maximum heights when air resistance can be neglected.

$$C_r = (\Delta h_{n+1} / \Delta h_n)^{0.5} \qquad \text{....... [1]}$$

## Rubber balls

Energy converted to heat in a rubber ball bouncing on tiles or concrete is mainly due to hysteresis in the rubber. Hysteresis loops for rubber plotted on a force compression graph increase greatly in area with the maximum compression of the ball and the coefficient of restitution is not expected to be independent of the impact energy. That this is the case for a regulation basketball dropped from a third floor balcony can be seen in figure 1.

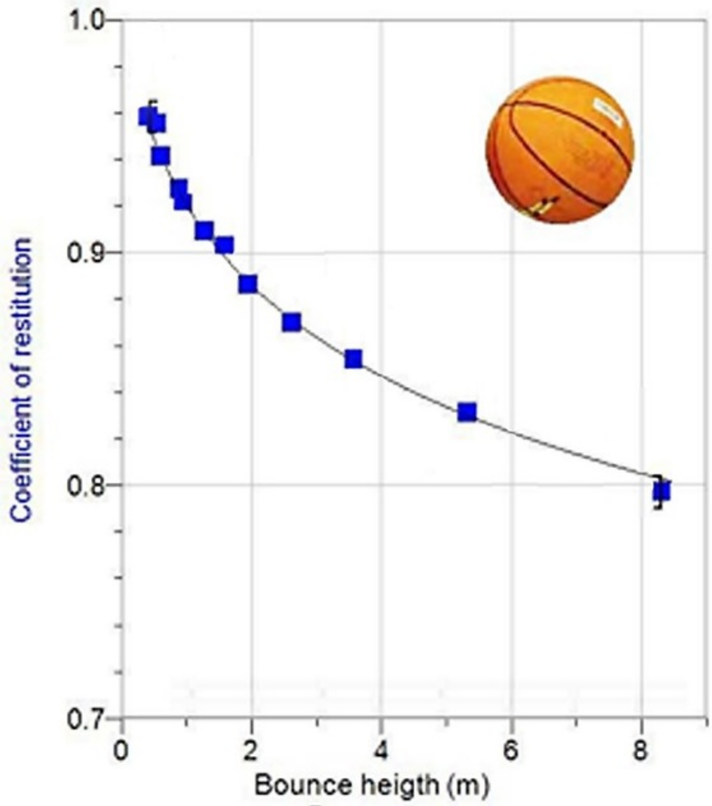



**Fig 1 –** approximate coefficients of restitution for a regulation basket-ball bouncing on concrete at 34°C.

**Note:** coefficients in figure 1 above were obtained from relative heights taken from video of the ball bouncing on concrete. Air resistance may have been significant and heights taken from video are subject to errors due to varying range. More accurate coefficients could be obtained from relative velocities before and after impact using video analysis or otherwise. The coefficient approaches 1.0 as bounce height reduces and the energy loss due to hysteresis becomes small. Determining how close $C_r$ approaches unity as bounce heights approach zero requires more data.

Unlike many plastics, metals and other solids the resilience of rubber increases with rising temperature in the zero to 100°C range. Most rubber balls, solid or inflated, have increased coefficients of restitution at increased temperatures.

Measurements at a range of temperatures can be made without a controlled environment by submerging the ball in a mixture of dry ice and alcohol and/or water from 0-100°C, using tongs to lift it out and immediately dropping it. If liquid nitrogen is used the polymer is likely to pass below the glass transition temperature and may shatter. Video analysis, motion detection, or sound recording, (see below), can be used to find the coefficient of restitution.

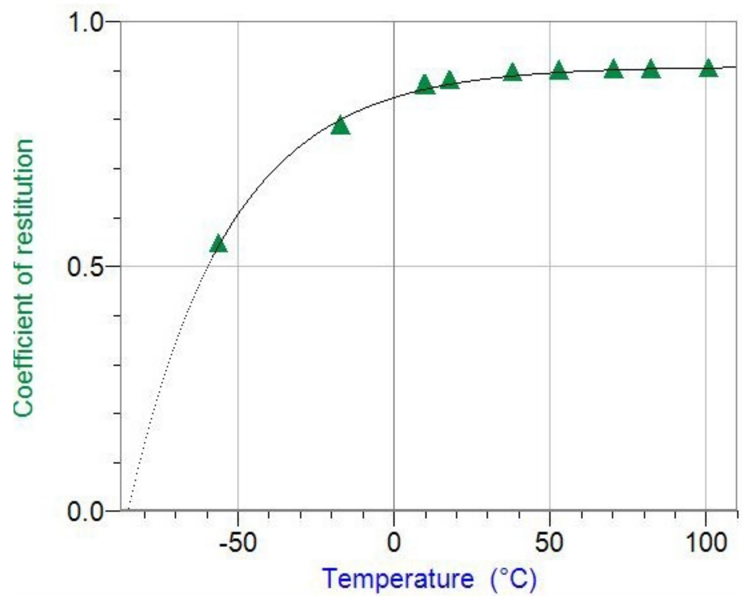



**Fig 2 –** typical coefficients of restitution with temperature for a solid rubber ball. Data: Yoshitaka Tamiya, ISB Journal of Physics, Vol, 4.

### Squash balls

Squash balls are slammed repeatedly against a wall before a game to warm them and increase the bounce height. The increased coefficient of restitution for warm squash balls is said to be due mainly to the change in the properties of the rubber, not to increased pressure inside the ball.

### Other balls

Pairs of solid balls of apparently similar black rubber are sold as "*sad and happy balls*". The happy ball is a normal neoprene rubber bouncy type, and the sad ball is made from polynorbornene (a polymer that has high hysteresis). Polynorbornene balls have a low coefficient of restitution at room temperature that *increases* as the temperature is lowered. Table tennis balls behave in a similar way. The coefficient of restitution increases as the temperature is lowered.

## DEMONSTRATIONS

### 1 A modified table tennis ball

A small hole is put in a 2.60 g table-tennis ball. The ball was bounced on tiles below a motion detector.

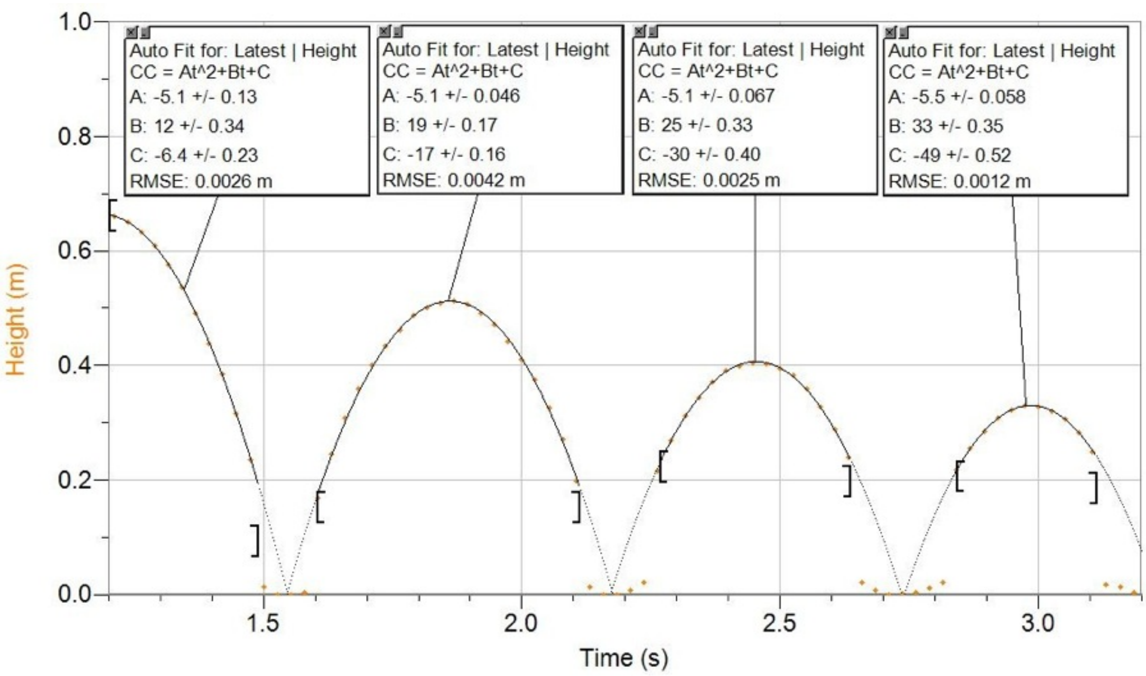



**Fig 3 –** a position-time plot for a bouncing table-tennis ball.

The coefficient of restitution reduces as an increasing mass of loose tissue paper is added to the table tennis ball through the small hole.

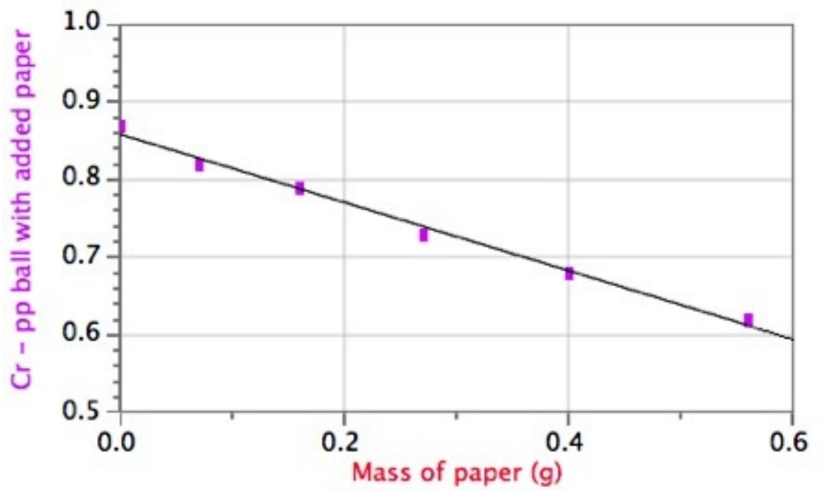



**Fig 4 –** Cr for a table-tennis ball versus the mass of tissue paper inside.

## An alternative method

$C_r$ is given also by the ratio of successive times of flight (see figure 3).

$$C_r = \Delta t_{n+1} / \Delta t_n \qquad \text{.......} \ [2]$$

The sound of a table-tennis ball bouncing on tiles clearly defines impact times. Vernier equipment allows a sample rate of up 100,000 points per second providing good time resolution down to bounce heights of two mm for a table-tennis ball bouncing on floor tiles.

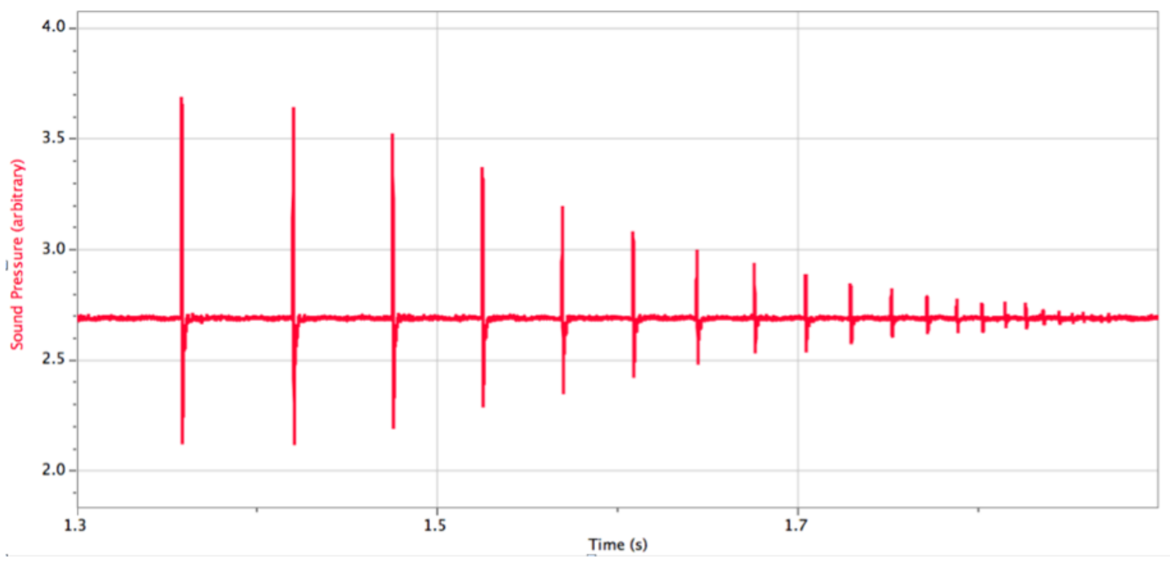



**Fig 5 –** a sound recording of a table tennis ball bouncing on tiles.

## 2 Playdough and slime balls

Playdough: a mixture of flour, salt, and water, is more resilient than clay.

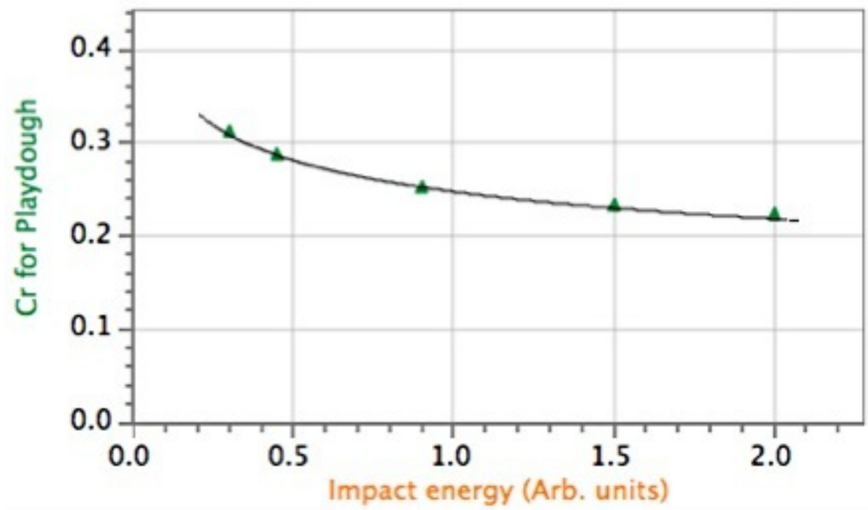
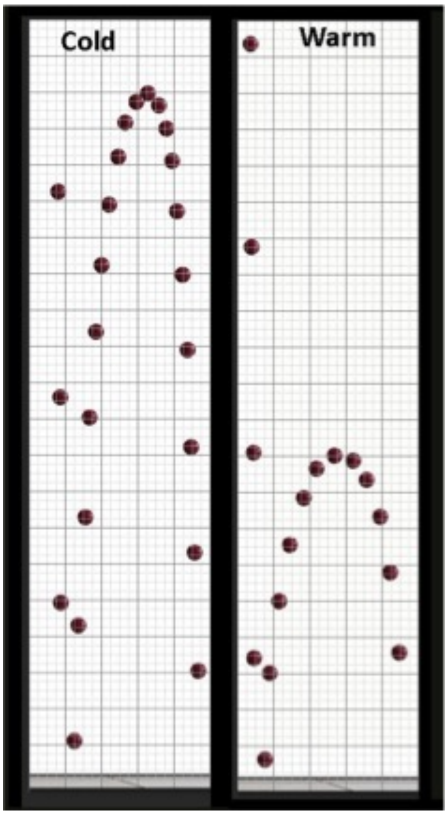



**Fig 6 –** the coefficient of restitution of a play-dough ball bouncing on tiles.

The coefficient of restitution for Playdough balls on tiles reduces with increasing impact energy in a similar way to $C_r$ for a rubber ball.

**Fig 7 – a** slime ball in a measuring spoon and bouncing at 5 and 34°C.




**Slime** can be made with borax and PVA glue or otherwise. If this home made slime is allowed to dry somewhat it can be formed into balls.

Putting a slime ball in the fridge increases the value of $C_r$.

*Plot a $C_r$/Temperature curve for home made slime.*

## Questions

**1** Prove that $C_r$ (the ratio of relative velocities before and after collision) equals 1 for any elastic two-body collision that conserves momentum and kinetic energy and prove that equation 2 holds for all balls bouncing vertically on level floors. (Neglect air resistance).

**2** How does the coefficient of restitution versus impact energy vary for a range of rubber and/or plastic balls (solid or hollow) on a hard surface. How close does $C_r$ approach 1.00 for low bounce heights?

**3** What is the temperature dependence of the coefficient of restitution for balls of a range of materials and construction, including table tennis and squash balls of different qualities?

**4** How does $C_r$ for table-tennis balls, squash balls and/or racquet balls, vary with and without a small hole, as a function of temperature?

**5** How does the coefficient of restitution of selected hollow balls (table-tennis ball, practice golf ball etc.) vary with the addition of a measured mass of loose paper, dry sand, or other materials.

**6** Use an acoustic method to compare the coefficients of restitution for bars and pipes of different metals bouncing vertically on tiles or other hard materials.

**7** Under what conditions (if any) does $C_r$ for balls of glass, rubber etc. depend on the radius of the bouncing ball?

**8** It is a said that $C_r$ for steel balls dropped onto the freshly cut stump of an Australian Blue Gum is remarkably high. Compare the coefficients of restitution for steel balls when dropped on selected local woods with different grain direction, sap and water content.