Curve fitting and power-law relationships

Introduction

Before curve fitting with computers and data logging with sensors and video and photograph analysis became possible, graphs were drawn by hand and the only curve that could be recognized was a straight line. Finding relationships from data required graphs to be linearized. When known relationships were verified in a school and undergraduate class, students were told what to plot. For instance, to verify that the period of a simple pendulum is proportional to the square root of the length, plot T^2 against l, which will be a straight line through the origin with a slope equal to $4\pi^2/g$. To find an unknown power law data was plotted on Log-Log paper.

These techniques are now of historical interest, but questions still do occur in public examinations. The graphs below plotted in a data logger illustrate what is (was) required.
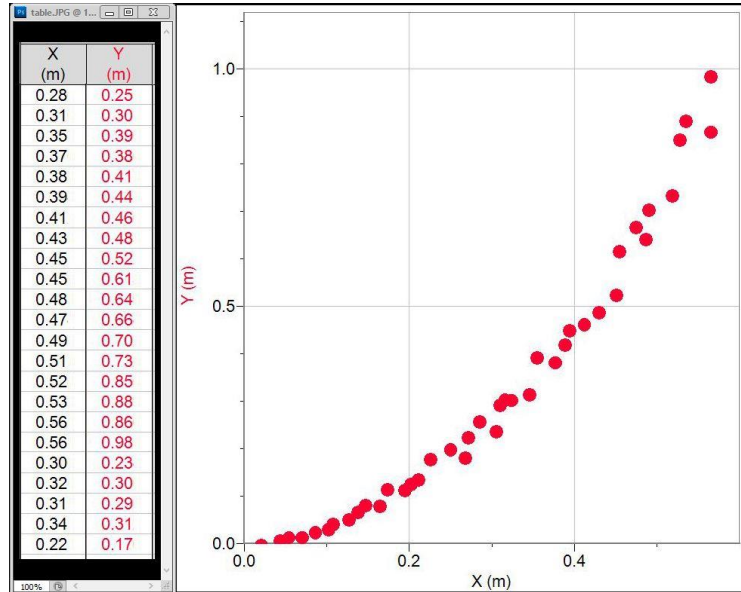
Washers are launched with initial velocities proportional to x and come to rest at y distances that depend in some way on the initial velocity. The y data is subject to random variation because the floor is not strictly uniform.

**Fig 1 – the** washers came to rest on a curve.

Examples

Inserting the photograph (figure 1) into Logger Pro and marking axes and positions gives the data plot below.

| X (m)	Y (m)
0.28	0.25
0.31	0.30
0.35	0.39
0.37	0.38
0.38	0.41
0.39	0.44
0.41	0.46
0.43	0.48
0.45	0.52
0.45	0.61
0.48	0.64
0.47	0.66
0.49	0.70
0.51	0.73
0.52	0.85
0.53	0.88
0.56	0.86
0.56	0.98
0.30	0.23
0.32	0.30
0.31	0.29
0.34	0.31
0.22	0.17

Graph 1 – the washers came to rest on a curve.

A computer generated power-law fit, $y = Ax^B$, shows at once that within errors $B = 2.00$ and the curve is a parabola.
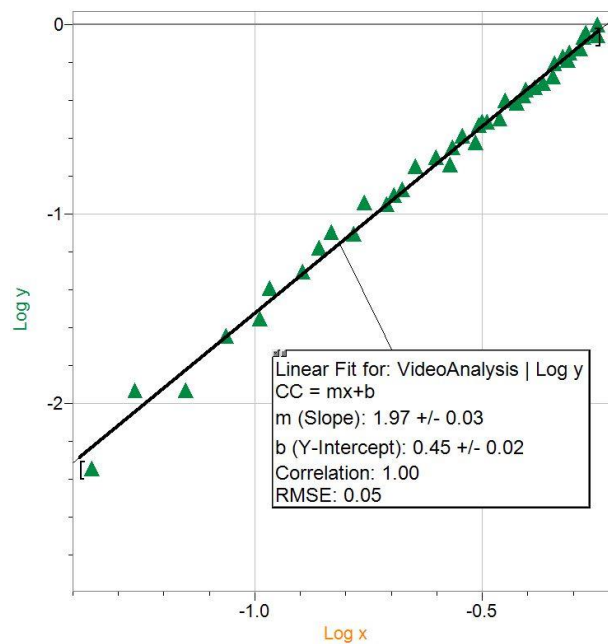

Auto Fit for: VideoAnalysis | Y
Y = AX^B
A: 2.94 +/- 0.12
B: 2.03 +/- 0.05
RMSE: 0.03 m
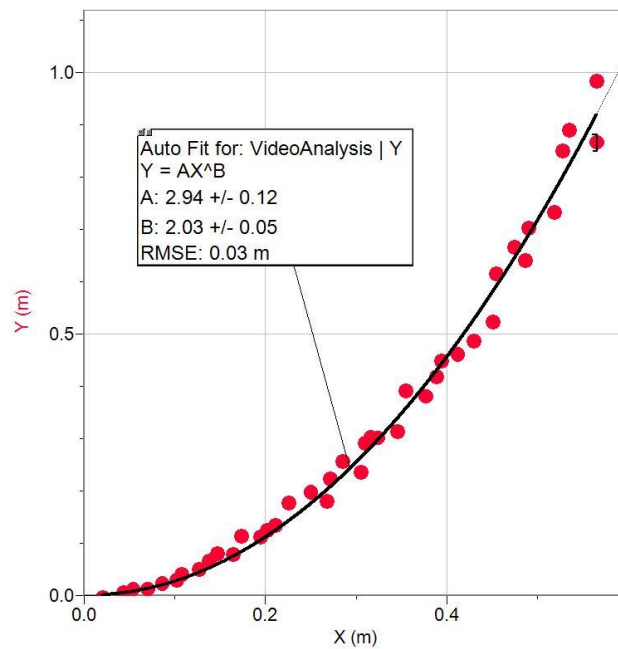
Graph 2 – the relationship is $y = 2.9\, x^2$.

Verifying that the curve is a power law with B = 2.00 by the old method involves plotting y against x^2.
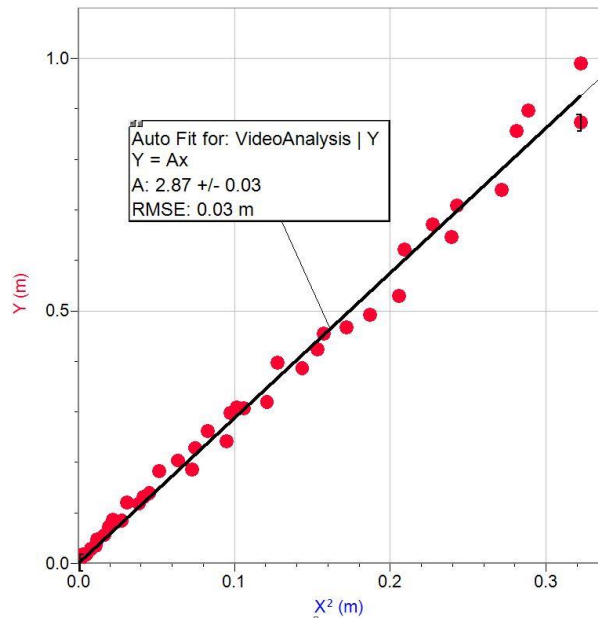
Graph 3 – the relationship is y = 2.87 x² as above.

The power law dependence can also be disclosed with a log y vs. log x plot.

Historically the last process was done by plotting x and y on Log-Log graph paper. Log-Log paper can be downloaded and the data in the table with Graph 1 can be plotted by hand. *That is something all of us should do at least once.*