Density invariant properties of liquids

1 Velocity of efflux

The velocity of a fluid that flows from a small hole in an open tank is given by …

$$v = \sqrt{2gh}$$

…where h is the depth of the hole below the surface … (when the effects of viscosity and surface tension are small and can be neglected.)

The equation is due to Torricelli. It shows that ejection velocities will be lower on the moon, but the velocity of mercury and water flowing from identical tanks in the laboratory will be the same, neglecting secondary effects due to surface tension and viscosity. We do not have a flask of mercury, but a saturated solution of zinc chloride has a specific gravity of around 1.8 and the viscosity and surface tension (see example 3 below) are increased by only moderate amounts by the presence of the salt in solution.

Figure 1 shows a large syringe barrel filled with water that flows from a small round hole near the bottom.

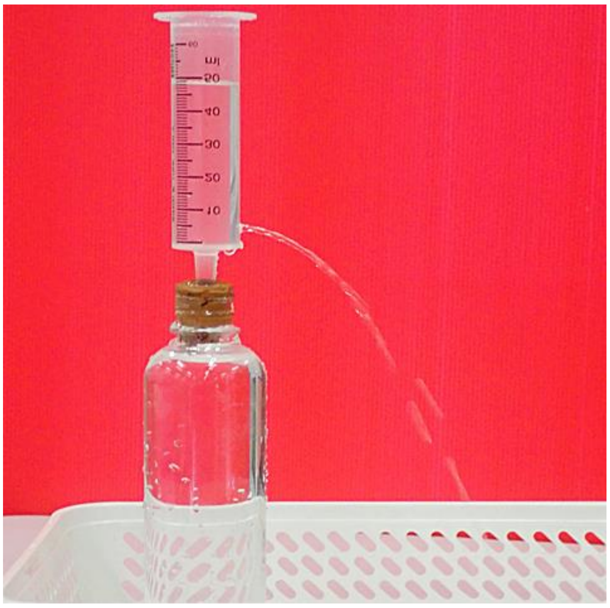

Fig 1 - water flows from a small hole.

For demonstration purposes two graduated cylinders with holes at the same depths are set up side by side and filled with water and zinc chloride solution. Removing the plugs (state skewer points) at the same time shows at once that the range over time of both streams is the same within close limits and that the time to empty the cylinders is the same. If a single cylinder is used, measurements may be taken from photographs or video.

Water depth above the hole and range data are plotted below.

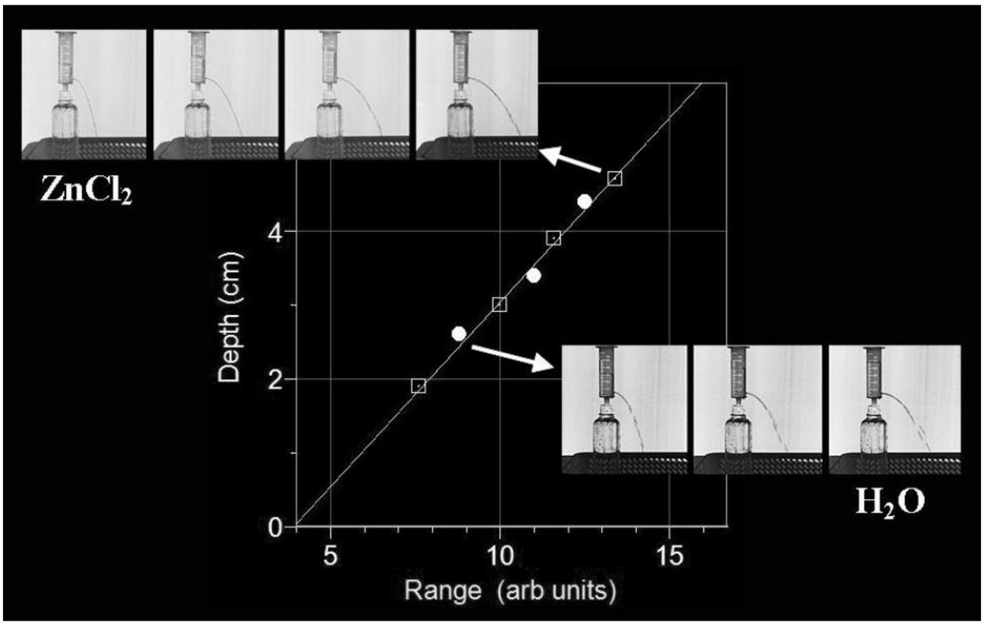

Fig 2 – water depth versus range for water and concentrated zinc chloride solution.

The inset images are greyscale inversions prepared from originals in the style of figure 1. Depth over range is the same for both solutions and velocity of efflux is independent of density. Very often in physics simplifications can be made by neglecting small effects like changes in viscosity and surface tension. Figure 2 shows that, in this case, Torricelli's equation holds within errors of ±5%.

In physical terms, water is driven from the tank by pressure. Pressure is proportional to the product of depth and density. The acceleration of a body of water close to the hole is inversely proportional to density and the result follows. A second demonstration of a similar invariance is described below.

2 Cavitation

When a 1.5 cm steel ball falls into water from a height of about half a metre a crater is formed at the surface with some ejection of water. The ball then opens up a cylinder of air through the body of the liquid. The cylinder of air soon pinches off a short distance below the surface and a bubble of air attached to the ball descends with it towards the bottom. The upper cone of air rapidly collapses, ejecting a return jet of water upwards. If plastic chips are added to the water regions close to the vertical cylinder of air show very little evidence of turbulence. The depth of pinch-of is constant to within 10% over many trials but the collapse of the upper open cone that follows pinch-off generates considerable large scale turbulence near the surface and the return-jet is unpredictable in height and appearance.

At higher impact velocity (following a drop approaching one metre) the cylinder may pinch off several times at roughly equal intervals in a deep tank leaving spherical bubbles of air to rise to the surface.

In a joint project KVIS and ISB students took 400 fps video clips of a steel-ball entering water and zinc chloride solutions of varying densities, under carefully controlled conditions. They confirmed the variable, unpredictable, nature of the return jet but found that the pinch-off was at the same time after entry and at almost the same depths in all clips. The frames below show pinch-off in water (left) and zinc chloride solution (right) with a specific gravity of 1.6.

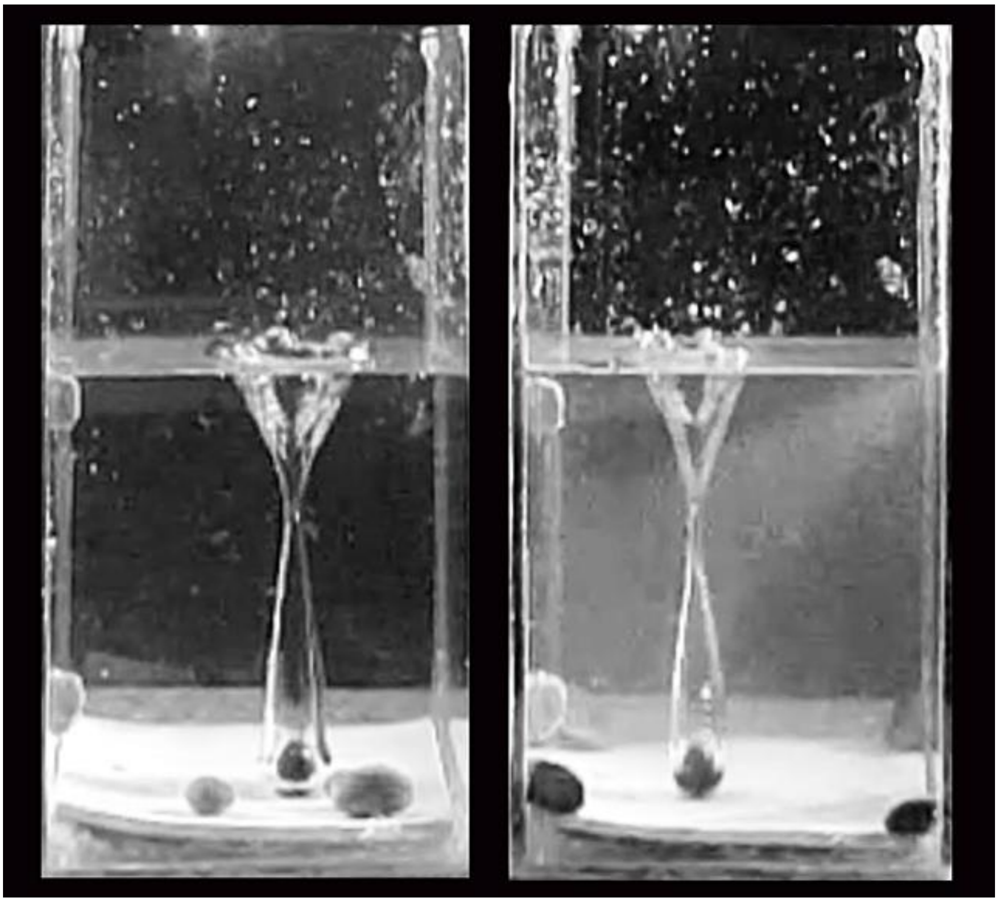

Fig 3 – pinch-off in water (left) and concentrated zinc chloride solution (right).

The similarity of the 29th frames after impact from 400 fps clips show that both the depth and time of pinch-off in water and zinc chloride solution (s.g. 1.6) are essentially the same. As above, pressure is the dominant effect and pinch-off is independent of density for the same reason. Pressure is proportional to the product of density and depth and the inward acceleration of water outside the cylinder wall is inversely proportional to the density. Looking at frames 28 and 29 in a series of video clips shows that the diameters of the collapsing air cylinders at the surface and elsewhere are consistently reduced in the zinc chloride by ~20%. It is suggested that the reason is the influence of increased surface tension.

Example 1 above is easily done as a lecture demonstration but this second example would be better presented with prepared high-frame-rate video clips. KVIS has a higher resolution high-frame-rate camera. Students will be asked to make and post clips as time permits.

Deep water gravity waves

Ocean swells and the ripples that expand from the impact site when a large stone falls into water are called gravity waves because the restoring force that returns displaced water to equilibrium is the weight of water displaced vertically. In physical terms the speed of gravity waves will depend in some way on the value of *g*. If oceans of water existed on Jupiter swells would travel faster than on earth. The speed depends also on the wavelength. Gravity waves are said to be *dispersive.* Notice that longer wavelengths are on the outside of the large expanding ripple pattern in figure 4.

Fig 4 – longer wavelengths travel faster.

Dispersion sorts waves from distant storms at sea by wavelength. Longer wavelengths arrive on shore first. On the eastern seaboard of North America the arrival of very long wavelength swells of large amplitude can indicate the presence of a hurricane hundreds of kilometres out to sea.

The restoring force that drives gravity waves is proportional the density of the water and the acceleration of a body of water is inversely proportional to density. The speed of gravity waves is independent of the density and waves of the same wavelength travel at the same speed on salt water and fresh water.

When water is deeper than half the wavelength (λ) the interaction of moving water with the bottom is minimal and the speed of gravity waves is given in the deep water approximation as …

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

[Derivations of this equation, and of the shallow water approximation $v = \sqrt{gd}$, that applies when wavelengths that are more than 20 times the depth, are on the web for those with the mathematical skills to handle second order partial differential equations.]

3 Wave velocity and density

The wave equation $v = f\lambda$ applies to all waves. The velocity of the components of a standing wave are easily found as $f\lambda$ if the frequency of the wave generator and the wavelength are known.

To demonstrate that wavelength is independent of density standing waves of the same frequency can be set up on liquids of different density. Improvised standing wave tanks are shown below.

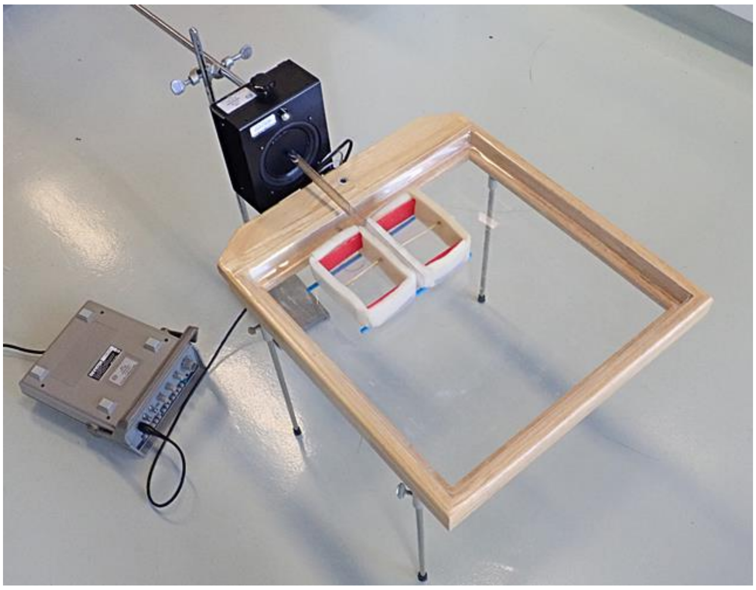

Fig 5 – ripple tank, speaker, signal generator and wave tanks.

A signal generator drives the small back and forth setting up standing waves in the boxes.

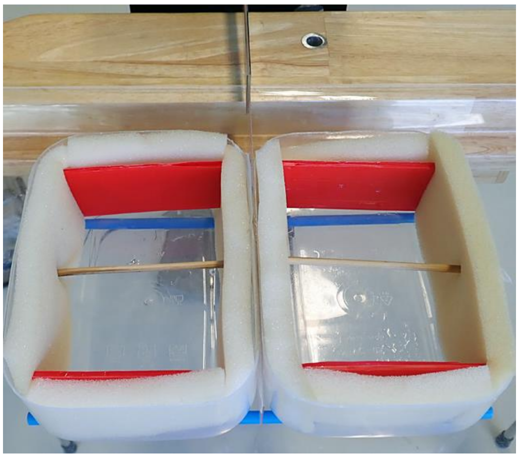

Fig 6 – standing wave tanks.

Water is placed in the tanks that have been modified with foam rubber and plastic board to make them rectangular, the same length, and to reduce wave generation at the sides. Rollers reduce friction between the tanks and base.

Two centimetres of water was placed in both tanks and the apparatus was put in the sun. Standing wave patterns observed on paper below the tank are photographed below.

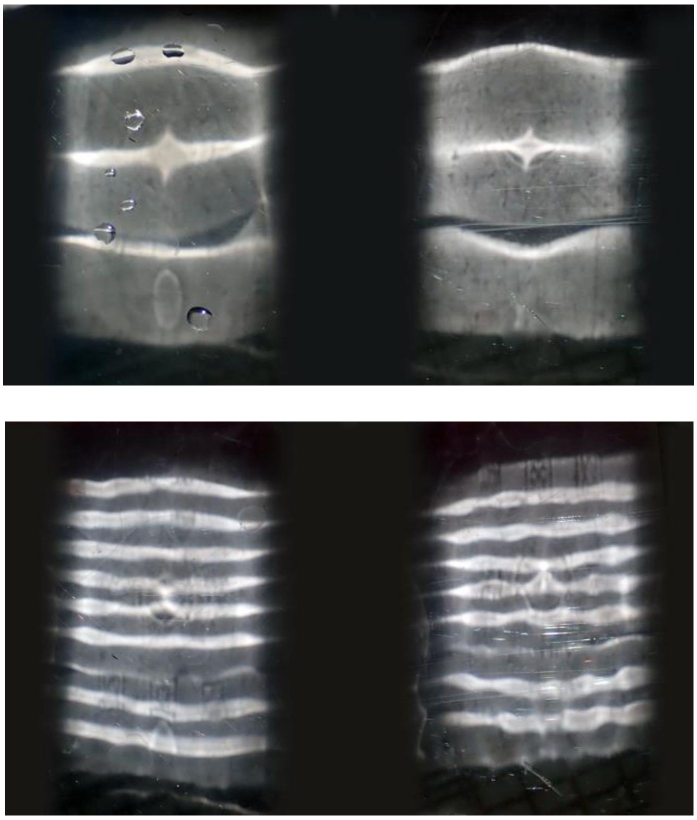

Fig 7 – standing waves on water.

The wavelength is the same in each tank as expected. At a low oscillation frequency (f_o) the wavelength is ~3 cm. At a higher frequency ($3f_o$) the wavelength is reduced to ~1 cm. The apparently weak lines in the lower image are due to the shadows of the spacers.

The tank on the left is now emptied and the water is replaced with a concentrated zinc chloride solution with a specific gravity of 1.6. There are two points to consider. Wavelengths of less than 2 cm become significantly influenced by surface tension which adds to the restoring force. At wavelengths of 2 mm or less (known as *capillary* waves) surface tension dominates. The speed of capillary waves rises as wavelength is further reduced. The dispersion of capillary waves is said to be anomalous with shorter wavelengths on the outside of ripple patterns generated by raindrops.

Because the surface tension of the zinc chloride solution is higher than that of pure water, relatively long wavelengths are required to demonstrate the independence of gravity-wave speed on density, remembering that the deep water approximation begins to break down for wavelengths that are more than half the depth of the water. The compromise situation is shown in figure 8.

In figures 8-10 zinc chloride is on the left and pure water is on the right.

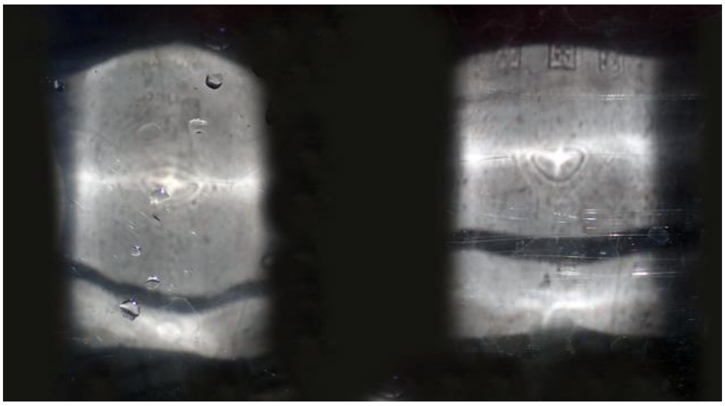

Fig 8 – at low frequency the wavelengths are the same at ~4 cm.

The wavelengths are identical above but as the frequency is increased the gravity-wave approximation begins to break down and the difference in surface tension of the two liquids begins to be important.

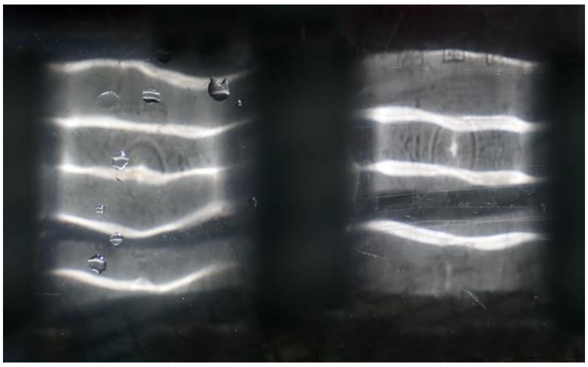

Fig 9 –at slightly higher frequency with wavelengths close to 3 cm.

In figure 9 increased surface tension is beginning to reduce the wavelength on the zinc chloride solution. Increasing the frequency again by a factor of 2 halves the wavelengths to about 1 cm and the difference in wavelength is more pronounced.

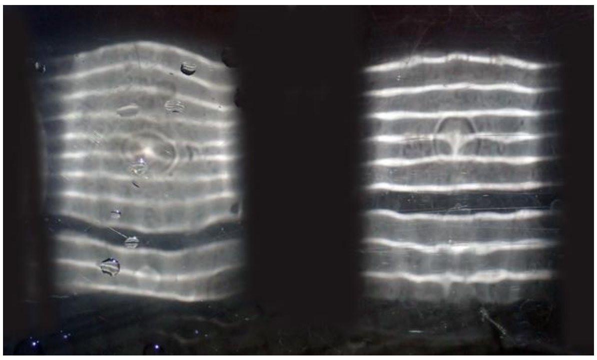

Fig 10 – the influence of surface tension increases at wavelengths close to 1 cm