Errors are not Mistakes

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Mistakes

In everything we do we try to avoid mistakes. Mistakes in exams cost marks and destroy scholarships. In life: mistakes are expensive.

Fig 1 – mistakes.

Errors

Errors are unavoidable uncertainties in measurements. They are not mistakes. For example: suppose we want to find the diameter of a bottle cap (figure 2).

Get a ruler with a mm scale, read the scale carefully to three figures, and write down …

D = 3.23 inches

That was a mistake. We should have written …

D = 3.23 cm

Now: suppose we give 12 friends a bottle cap and a ruler. We ask them to do what we just did.

Fig 2 – a bottle cap and a cm scale.

The numbers look like this …

3.25
3.23
3.23
3.24
3.22
3.23
3.25
3.25
3.26
3.24
3.22
3.25

Could they all be right? Could they all be wrong? We can find an average by adding and dividing by 12 ….

D = 3.24 cm

… but we still haven't got this right.

Our friend's average number is not the same as our number. Do the bottle caps have different diameters? Are the bottle caps not round? Are different people reading the rulers differently? Are the rulers not all exactly the same? The differences are random errors. Random errors occur every time we try to measure something carefully. To deal with this we write measurements with a plus or minus sign.

$$\mathbf{D = 3.24 \pm \Delta D}$$

The random error, ΔD, is the uncertainty in D. In experimental work you will always write measurements this way. Finding the right value for ΔD often requires experience and good judgment. *If you're not sure: ask*. We will outline three ways of finding ΔD for the diameters in the list above.

Method 1: half the range (4 or fewer values)

Subtract the smallest value from the largest to find the range.

3.26 – 3.22 = 0.04

We write … D = 3.24 ± 0.02 ... where 0.02 is half the range.

Method 2: average deviation (5-8 values)

Look at the table and write down the deviations (the positive difference between the average and the number in the table …

0.01, 0.01, 0.01, 0, 0.02, 0.01, 0.01, 0.01, 0.02, 0, 0,02, 0.01

The average deviation is 0.13/12 = 0.01 to one significant figure.

We write ... D = 3.24 ± 0.01 ... where 0.01 is the average deviation.

Method 3: standard deviation (SD) (8 or more values)

Standard deviation can be calculated in Logger Pro, (or found by noting the half-width of the distribution at half-height). Two out of three values in a normal (Gaussian) distribution lie within one standard deviation of the mean.

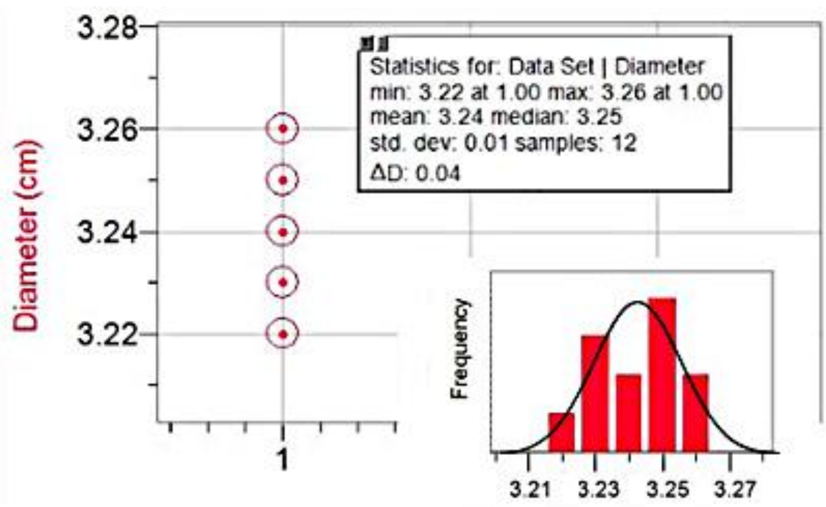

Fig 3 – The histogram and Gaussian fit.

We write … D = 3.24 ± 0.01 ... where 0.01 is the standard deviation.

The calculated standard deviation has been rounded from 0.013 to one significant figure. For a small number of measurements the distribution will be only an approximate fit to a Gaussian function as above. The method used to find the estimated error should be stated in lab-reports and papers.

The Central Limit Theorem

The *Central Limit Theorem* lies at the foundation of statistics and explains why the Gaussian probability distribution is common enough to be called the *Normal* distribution. In simple terms the theorem states that: -

> When independent random variables are added, their sum tends toward a Gaussian distribution (commonly known as a *bell curve* or *normal distribution*) even if the original variables themselves are not normally distributed. In other words: data that are influenced by many small, unrelated random effects (like the height of boys at KVIS or the mass of a can of beans) are approximately normally distributed.

Wikipedia has a discussion and references. (The History link has names, dates, and details). https://en.wikipedia.org/wiki/Central_limit_theorem#History

The normal distribution of random errors

In the example above the distributions of individual errors may not be Gaussian, but the sum of the many smaller contributions *is* Gaussian. The point is important because it shows that a standard deviation can be used as a measure of likely error and it allows a deeper analysis of the propagation of random errors than normally done in elementary physics courses. (See: *Error Propagation*.)

Systematic errors

Random errors can be estimated from a set of independent readings with the methods described above, but if it is not possible to repeat measurements, random errors must be estimated from the physical situation: a more difficult process even for experienced scientists. *Systematic errors* that affect all measurements to the same extent cannot be found by repeating measurements. For example: suppose that a five metre tape-measure is made of steel and calibrated at room temperature. If measurements of ~5 m are made at -40 ºC there will be a small systematic error because all the tapes will be shortened by the same small amount. This temperature dependent *systematic error* cannot be found or reduced by repeating measurements and must be estimated from the known value of the coefficient of thermal expansion of steel.

The accuracy of measuring instruments

As a first estimate, analog instruments *in good working order*, are assumed to be accurate to within half the smallest scale division over their full range. A metre long mm scale is expected to be accurate to within ± 0.5 mm when measuring things that are close to a meter in length. When analog instruments were used to measure current, voltage, temperature etc. half the smallest scale division was a common convention.

This simple convention does not apply to digital meters and manufacturer's manuals must be consulted for their rated accuracy over different ranges.