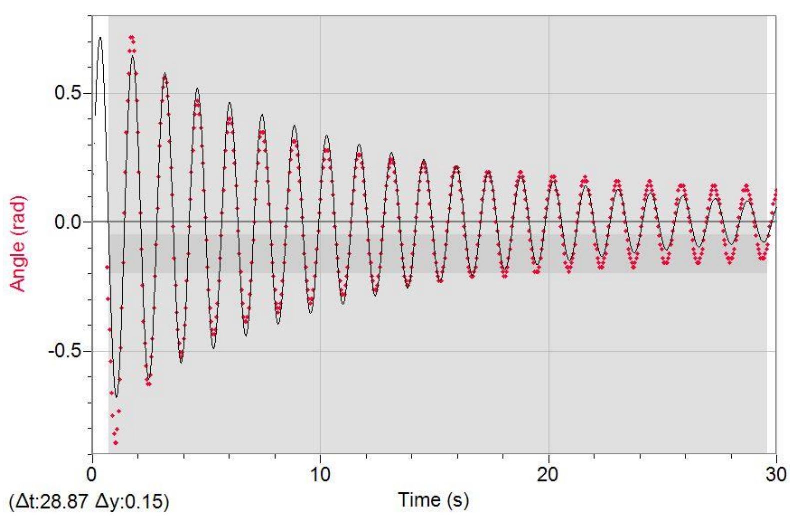
Exponential decay

A counter example

There is a common belief that the damping of the motion of a pendulum in air is exponential, or nearly so, in all situations. To explore the limits of that approximation we attach a rod and polystyrene ball to a Vernier angular motion detector.

Fig 1 – a rod and ball pendulum

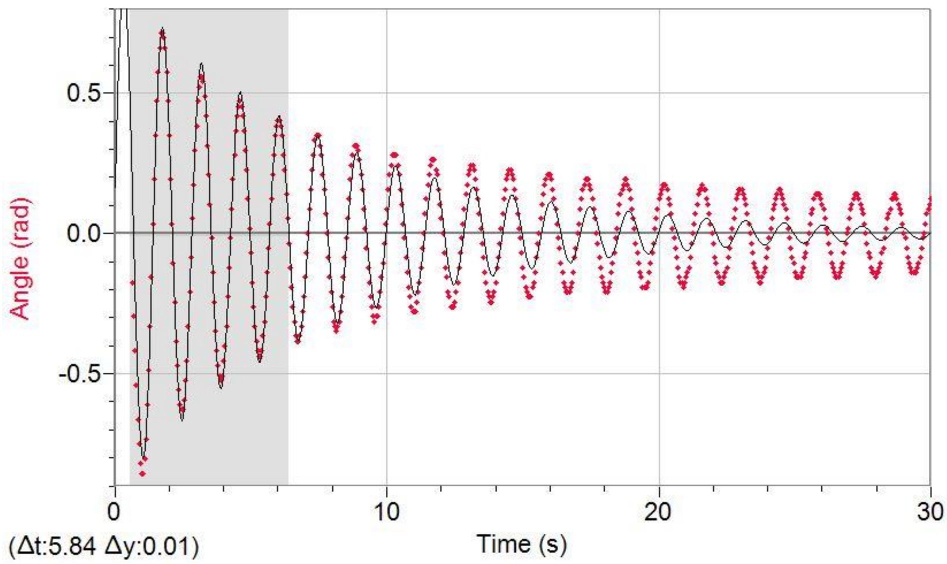
At a first glance the amplitude decay appears to be exponential, but an auto-fit that takes all points into account is a close fit only over the central five or so periods.



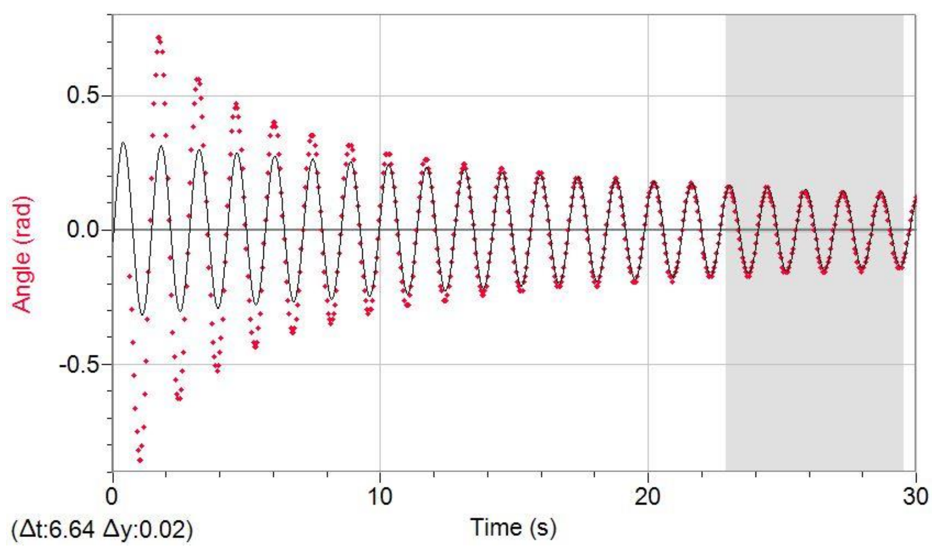
Graph 1 – amplitude decay with an exponential decay fit.

The deviations in amplitude over 30 periods rise to more than ±20%.

Auto-fitting a sine function with exponential amplitude decay to the first 5 periods (Graph 2) and the final five periods (Graph 3) shows that exponential decay is not a good approximation in this situation.



Graph 2 – amplitude decay with an exponential decay fit.



Graph 3 – amplitude decay with an exponential decay fit.

The retarding force due to air resistance in turbulent flow is proportional to velocity squared. The evidence suggests that this is the dominant component of the damping force in this situation.

Exponential decay: example 1

Modifying the pendulum by replacing the ball with a flat plate may give a drag force that depends on viscosity (is proportional to velocity) provided drag due to turbulence in the air and frictional torque in the angular motion detector can be neglected.

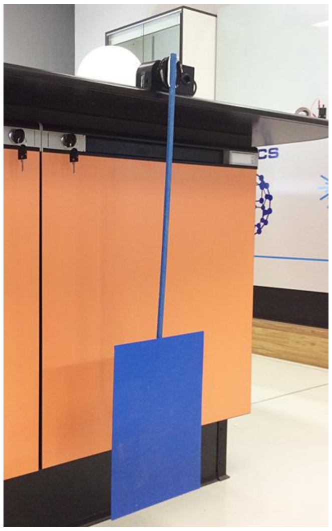
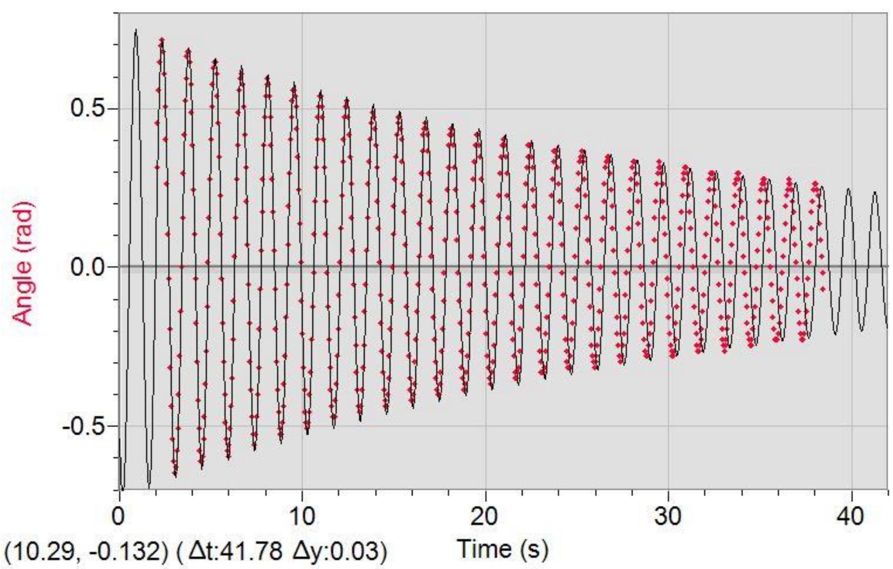


Fig 3 – a rod and vane pendulum

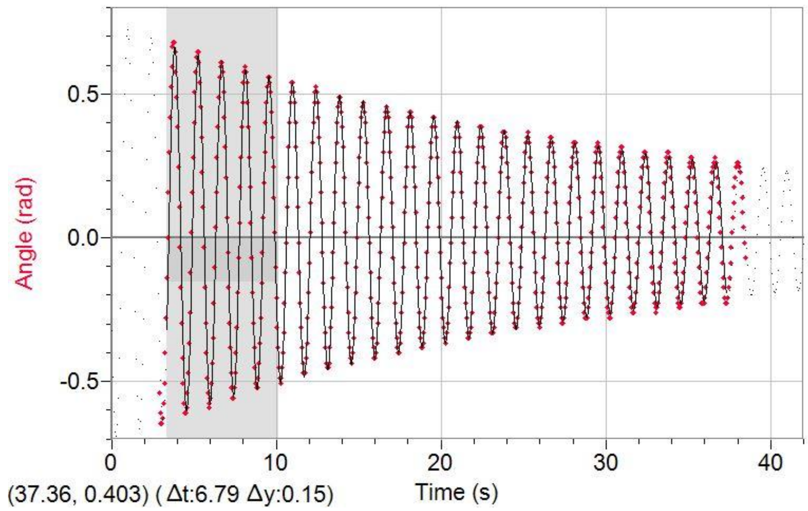
Amplitude decay for this pendulum with an auto-fit including all points as above is shown below.



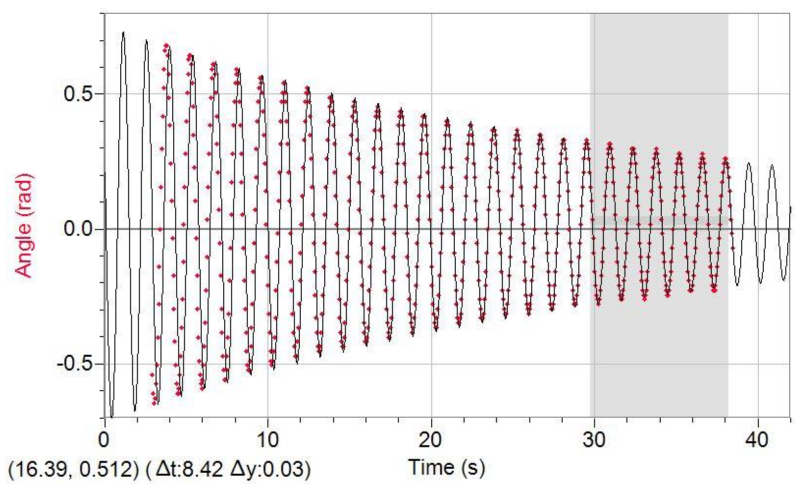
Graph 4 – amplitude decay with an exponential decay fit.

The curve fit is very much closer than the corresponding fit when using the ball.

Again auto-fitting a sine function with exponential amplitude decay to the first 5 periods (upper graph) and the final five periods (lower graph) shows that the exponential decay model applies within close limits over the full range of 30 periods.



Graph 5 – amplitude decay with an exponential decay fit.



Graph 6 – amplitude decay with an exponential decay fit.

The curve fits (Graphs 4-6) provide evidence that the damping due to the motion of the vane through the air is dominated by viscous drag, not by drag due to turbulence.

Dashpots

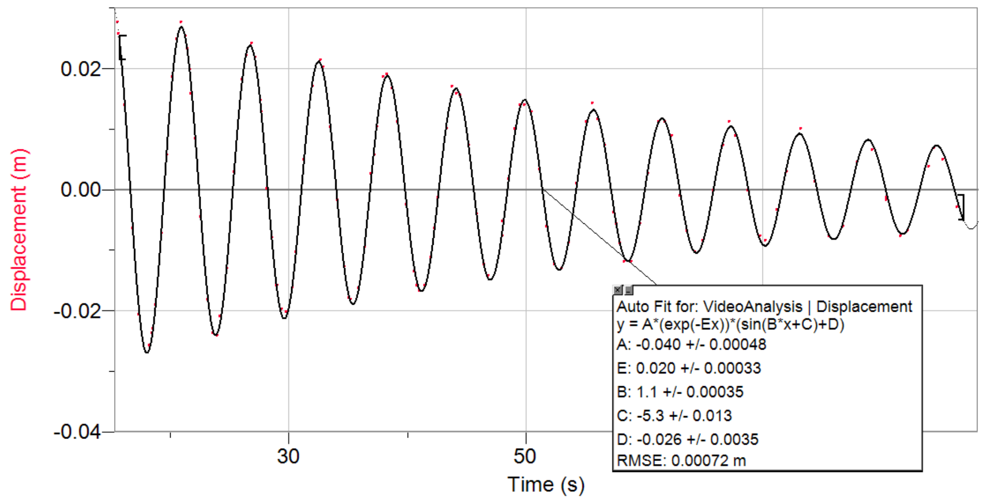
In many situations an oscillation is damped by a dashpot. There are many forms, some rely on rotation and some on straight line motion, but all involve the motion (for example of a loose fitting piston in a cylinder) in a medium such as oil or air. Drag force is approximately proportional to velocity and dashpot-damping is near exponential.

Exponential decay: example 2

Water rests in a 5 cm internal diameter tube in the form of a circular arc. When displaced the slug of water oscillates about the equilibrium position with simple harmonic motion.

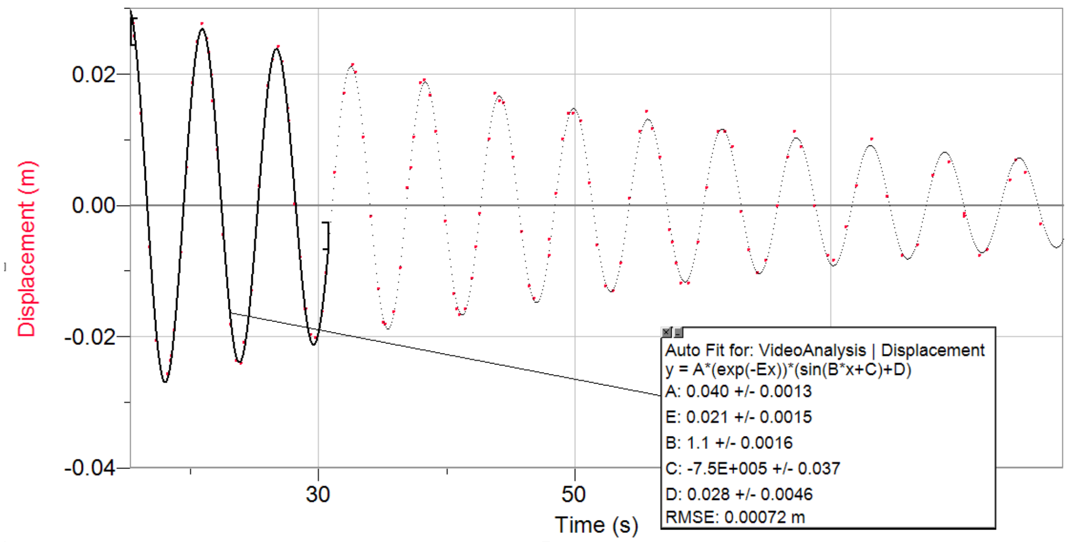
Fig 1 – water in plastic tube.

The water motion is in the transition region between laminar and turbulent flow. Adding salt to the water to suspend plastic chips and mark the flow shows that water moves as a whole over most of its length with localized turbulence within 3 cm of each end. If damping is dominated by viscous drag the amplitude of oscillations will follow an exponential decay function because the equation of motion when the retarding force is proportional to velocity has an exact solution: a sine function multiplied by an exponential decay factor.

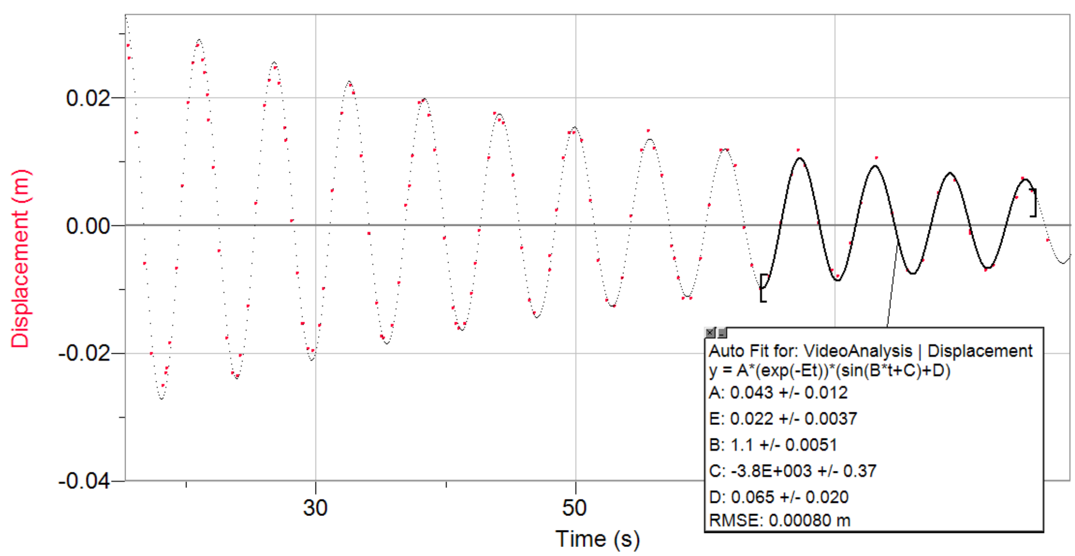


Graph 7 – data points from video analysis with an auto-fit of a sine function with exponential amplitude decay.

The exponential decay function in graph 1 appears to be a good fit to the data points over the 12 periods shown. To confirm the exponential decay model graphs 8 and 9 below show fits over restricted period ranges (bold lines).



Graph 8 – an auto-fit of a sine function with exponential amplitude decay.



Graph 9 – an auto-fit of a sine function with exponential amplitude decay.

Graphs 8 and 9 show that exponential amplitude decay is a satisfactory model in this case, indicating that viscous drag, which is proportional to velocity, is the dominant damping mechanism in this situation.