Figure 1 Royal Palace, Bangkok

Fractals in Thai Cultural Designs

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Abstract

A fractal is a geometric shape displaying a never-ending pattern of high complexity with the property of self-similarity at many levels, and one that is generated by iterations of simple equations or figures. Fractals started to be studied to describe chaos by using equations and graphs just a few decades ago but quickly sprung out of classrooms to everyday life and became a fashionable object on its own used in pop culture, graphics, and arts. Many fractals include repetition of a triangular- or rectangular- or some other geometric shape. In this paper we try to showcase the beauty of traditional Thai Cultural Designs and point out their fractal-like structure that can be found in typical Thai designs hundreds of years old, a long before even the word "fractal" was defined. We look at many examples of temples, pagodas, stupas, tiered umbrellas, shrines, sculptures, paintings, ornaments etc., explain the patterns of those designs and identify a few types of fractals commonly found in those places. We classify them and describe their shapes, level of self-similarity, and complexity. Some of these designs have been defined mathematically by finding a generator for a simple iterative process.

Keywords: Fractals, Thai culture, Design

Introduction

For a tourist there is an abundance of beautiful sights in Thailand. But for a mathematician there is an… abundance of beautiful Fractals in Thailand! Here we are going to look at many famous landmarks in Thailand and show that they have mathematical fractal-like structure.

A fractal is a never-ending pattern with self-similarity at different scales. It can be created by repeated iterations of a figure or of an equation over and over. It lends itself particularly well to describing nature and real life phenomena [1, 2] but also architecture [3]. The word "Fractal" was first used in 1975 by Benoit Mandelbrot, who studied set of points in complex plane that stay in the set and satisfy the equation:

$$z_{n+1} = z_n^2 + c$$

Where z_{n+1}, z_n, and c are complex numbers, $z_0=0$, z_n is currently used number, z_{n+1} is the result of the calculation for $n = 0,1,2,3,\ldots$ and c is a constant. The calculation is repeated over and over. When Mandelbrot iterated the above function over all the points c in complex plane he came up with this, famous now, image:

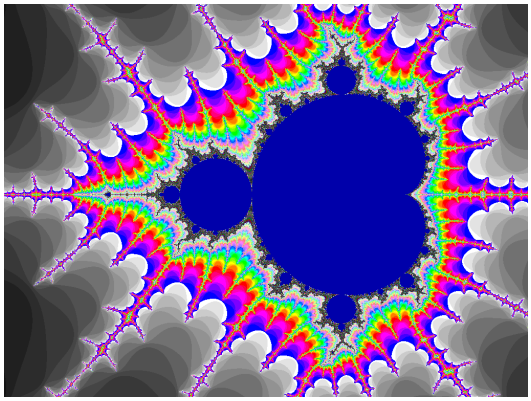

Figure 2: Mandebrot Set

One can see similarity of the Mandelbrot Set and many of the Thai designs like roofs, chats, stupas, and others. Below we consider the elements of the design that make them fractals.

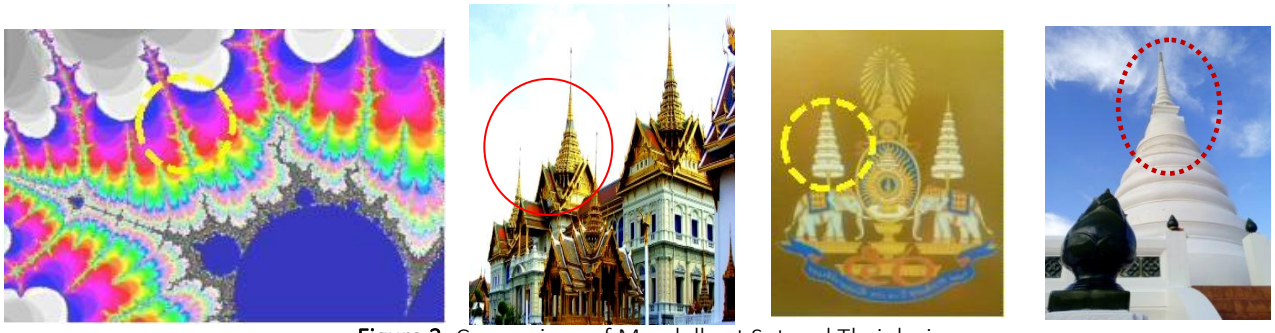

Figure 3: Comparison of Mandelbrot Set and Thai designs

Materials and Methods

1. Collect pictures of Thai cultural design of many places in Thailand; temples, pagodas, stupas, tiered umbrella, sculptures, ornaments, and etc. by searching on the internet and visiting the places.
2. Investigate and analyze the pictures for types of fractals they are, looking into how many complexity levels are there? What are their shapes? What are the generators? etc.
3. Describe and classify each picture in details.
4. Find fractal mathematical descriptions and those pattern, which can be constructed using equations.

Results and Discussion

There are many types of shapes and designs in Thai Culture but they can be divided into three general types of Thai Cultural designs: roofs of buildings, pagodas and stupas, and ornaments. We will describe each of them in detail.

Roofs of Buildings

Most roofs have two to four parts (from body to crest). Common shapes of the bodies are rectangle, hexagon, octagon, and dodecahedron. Number of iterations is usually a Natural number between two to six for each part. There are a variety of styles of the crests, sometimes consisting of several patterns, for example, triangles with Thai painting, squares with petal-like ornaments, and rectangles with flower-like structures. Figures 4 and 5 show examples and Table 1 shows more information and some of the places we were looking at.

Figure 4. Wat Prathatsuthone Mongkol Samakkeedham, Prae

Figure 5.Chakri Maha Prasat, Bangkok

Name and Location	Shape	Number of iterations
NoenSawang Temple, Rayong	Triangle	2
Pak Nam Temple (1), Rayong	Triangle	2
Pa Pradoo Temple, Rayong	Triangle	3
Pak Nam Temple (2), Rayong	Square	3
Phimai Rock-Castle, Nakhon Ratchasima (Phimai Historical Park)	Lateral stacking	5
Phanom Rung Rock-Castle,Burirum (Phanom Rung Historical Park)	Lateral stacking	5
ChakriMahaPrasat, Bangkok	Hexadecagon	6
DusitMahaPrasat, Bangkok	Hexadecagon	6
SanphetMahaPrasat, Bangkok	Hexadecagon	11
Rong Kun Temple, Chiang Rai	Square, Triangle	11
PrathatsuthoneMongkolSammakkeedham, Prae	Rectangle	21

Table 1: Shape and number of iterations of roofs of buildings

Pagodas and Stupas

Figure 6. Kuu Kham Temple, Chiang Mai

Pagodas and stupas contain two parts (body and crest). Common shapes of bodies are square, hexagon, and circle. Number of iterations is a Natural number from 3 to 18. There are a variety of styles of the crests, some are combined with other patterns. Tiered umbrella-like crests, in particular, have been found in majority of the places. Table 2 shows some of the designs we have looked at and Figures 6 and 7 show some examples.

Figure7. ChediKlang Nam, Rayong

Name and Location	Shape	Number of iterations
PrakaewdontaoSucharam Temple (1), Lampang	Circle	3
ChediKlang Nam (1), Rayong	Circle	3

Pra ThatLampangLuang Temple, Lampang	Circle	4
SuwanChedi (1), Lampoon	Square	5
Chedi Liam (Kuu Kam Temple), Chaing Mai	Square	5
SuwanChedi (2), Lampoon	Square	8
PrakaewdontaoSucharam Temple (2), Lampang	Circle	9
Rong Kun Temple, Chiang Rai	Square, Triangle	11
ChediPranaresuan, Pitsanulok	Circle	12
ChediKlang Nam (2), Rayong	Circle	18

Table 2: Shape and number of iterations of pagodas and stupas

Ornaments

Ornament forms were being studied on paintings, sculptures, and decorations, which are generally based on natural or Thai traditional designs. They consists of repetitions of various shapes in various sizes intertwined together to form an elaborate masterpiece. Below we compare some of the decorations to a detail of Mandelbrot Set. Some more examples are shown in Figures 9 - 13, and a list of types of ornaments is in Table 3.

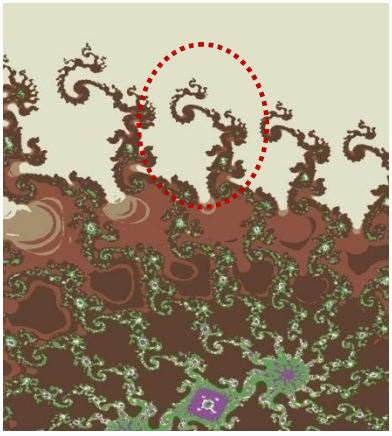

Figure 8. Detail of Mandelbrot Set

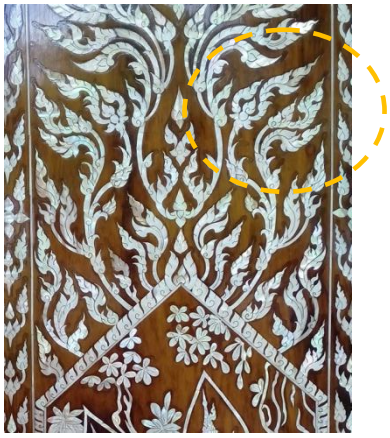

Figure 9 Guilded lacqured cupboard (3)

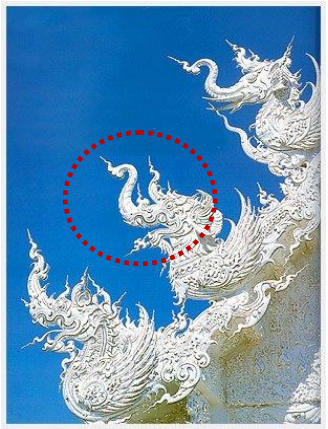

Figure 10. Wat_Rong_Khun, detail of roof

Name of Work	**Shape**	**Name of iterations**
Banana leaves floating basket (4)	Triangle	3
Tieredumbrella-like carving candle	Circle	5
A leaf made with cucumber carving work (1)	Oval	7

Flower-watermeloncarving work (1)	Circle petal-like	13
A leaf made with cucumbercarving work (2)	Rhombus	16
Dahlia-watermeloncarving work	Petal-like / (outside) Triangle / (inside)	4 4
Flower-watermeloncarving work (2)	Notched petal-like / (outside) Petal-like / (inside)	3 6
A window at NongPhaWah Temple, Rayong	Ring Rectangle Rectangle	5 4 3
An entrance of Pak Nam Temple, Rayong	Triangle / (1st body) Square / (1st top) Ring / (crest) Triangle / (2nd body) Rectangle / (2nd top)	3 4 4 2 4
Gilded blacklacquer cupboard* (1)	Tree-like	
Gilded blacklacquer cupboard* (2)	Water-like	
Gilded lacquer cupboard* (3)	Flame-like	

*The pattern of decoration is natural form, which is fractal itself.

Table 3: Shape and number of iterations of ornaments

Figure 11 Flower- watermelon carving work (2) **Figure 12** Gilded black lacquer cupboard (2) **Figure 13**. Loy Kratong Float decoration

Tiered Umbrellas

For the remainder of this work we focus on a decoration called a Chat or a Tiered Umbrella. A tiered umbrella is a religious and royal symbol, representing divinity or kingship, it is found distributed over king's attributes like throne, bed, funeral urn, etc. and in temples over images of Buddha. There are several tiers with either three, five, seven, or nine shelves. We have chosen tiered umbrellas for an interpretation of a Thai design fractals due to their common shape and simple iterations. The example of this decoration is shown in a royal emblem in Figure 14.and some other examples are in Table 4.

Figure 14. Royal Chat

Name of Place	Name of iterations	shape
Pa Pradoo Temple, Rayong	5	Tiered
NongPhawa Temple, Rayong	5	Tiered
Pak Nam Temple, Rayong	5	Tiered
Rong Kun Temple, Chiang Rai	5	Tiered
PrathatJomjo Temple, Chiang Rai	5	Tiered
PaDarapirom Temple, Chiang Mai	7	Tiered
PrasatDoisuthep Temple, Chiang Mai	7	Tiered
Pra Sing Temple, Chiang Mai	7	Tiered

Table 4: Shape and number of iterations of tiered umbrellas (Chats)

Many Thai designs and decorations have the property of self-similarity and can be expressed in terms of simpler shape that can be iterated and repeated many times over. Such shape can have fractional dimension.

Fractal dimension

In order to describe the dimension of a fractal we consider its generator, i.e. a simple shape transformation, that is then iterated many times over. Its dimension is calculated using number of elements of generator and their scale in the following way

$$\boldsymbol{D = \frac{logN}{logR}}$$

Where *D* is the dimension, *N* is number of pieces in the generator resulting from construction, and *R* is the linear magnification.

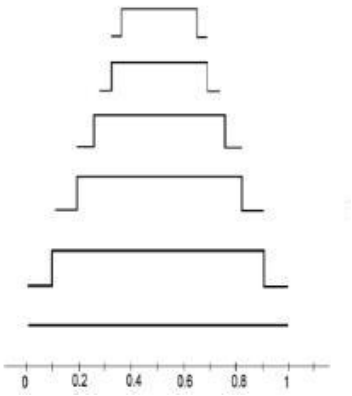

Fig. 15 Construction of simulated tiered umbrella (1)

Figure 15 is obtained from the line at the bottom that is divided into 10 equal parts and the middle 8 are replaced by 10 pieces as in the first figure (the generator) above the line. We used the following values: $N = 11.9$, $R = 10$, $D = 1.076$. Shown are 5 iterations of the transformation. The dimension of a fractal is usually a fractional number.

A matrix equation

Many major transformations of graphs, like translations (shifting), rotations, reflections, non-rigid transformations (scaling) can be represented by matrix equations. A simple 2-D transformation of scaling and rotation can be represented by a 2x2 matrix, and shifts can be represented by vectors.

In Iterated Function Systems (IFS) fractals are obtained by iterating a simple geometric figure according to a simple linear matrix equation. The diagram in Fig. 16 can be constructed by a series of affine transformations given by the matrix equation:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

Where, x_{n+1} and y_{n+1} are new horizontal and vertical coordinates respectively, x_n and y_n are current horizontal and vertical coordinates respectively, and *a*,*b*, *c*, *d*, *e*, and *f* are constants. Constants a, b, c, and d represent scaling, and e and f represent shifts.

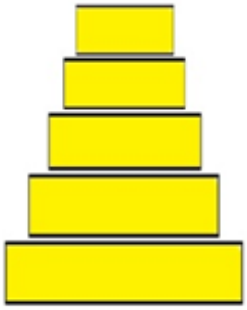

Fig. 16 Construction of simulated tiered umbrella (2)

Figure *10* is obtained by repetition (five iterations), scaling and shifting of the bottom most rectangle. According to matrix equation above, *a*=0.8, *b*=*c*=0, *d*=0.95, *e*=0.1, and *f*=0.095. The coordinate system Origin is set to be the lower left corner of the bottom rectangle.

Conclusions

Thai traditional Designs are rooted in Fractals. We have looked at different types of Thai Designs and described the types of fractals, their generators, their shapes, and numbers of iterations. We defined some of these designs using IFS functions and calculated their fractal dimension.

Acknowledgments

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