

## Demonstration: free fall

### Displacement intervals as the sequence of odd numbers

Washers are looped on a long string (strong cotton thread) at intervals that increase in the ratio of the odd numbers beginning with one. The first interval on the string is set at one cm so that 17 washers make a line close to 3 m long.



**Fig 1** - washers, tape, thread, a completed line and a metal tray.

Galileo described constant acceleration as motion for which the intervals traveled in set time intervals increased as the odd numbers. In modern terms  $s$  (the distance moved from rest) is equal to  $\frac{1}{2}at^2$  where  $a$  is the constant acceleration. *Can the reader show simply that the sum from 1 to  $n$  of the sequence of terms  $(2n-1)$  is equal to  $n^2$  for all  $n$ ?*

Small washers will stay in place but a spot of glue can be added if the line is to be kept for another time.

**Fig 2** - a washer looped on the line line.

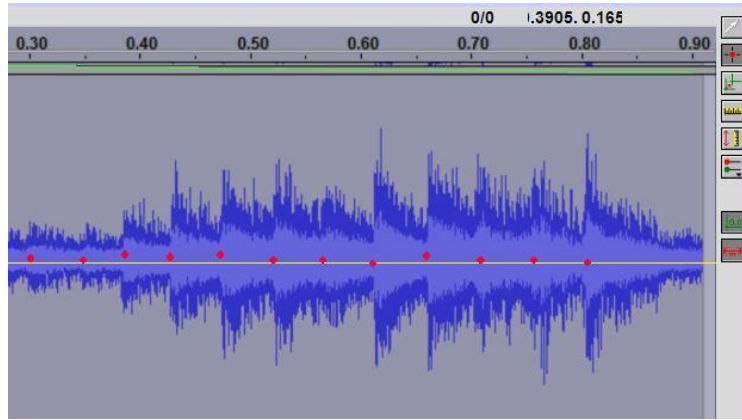


Note: attaching a washer on a line in the right place is not as easy as it sounds. Try extending the line along a long measuring tape, marking the position of the next washer ( $n^2$  cm) with a pen allowing for half the effect of the loop. The string is then pushed through the washer and looped as shown.

## The demonstration

*Hold the string steady with the bottom washer just touching the metal tray and carefully drop it. The impacts will be heard to be equally spaced.*

To make the demonstration quantitative and illustrate the image analysis feature of Logger Pro record the drop in Audacity.

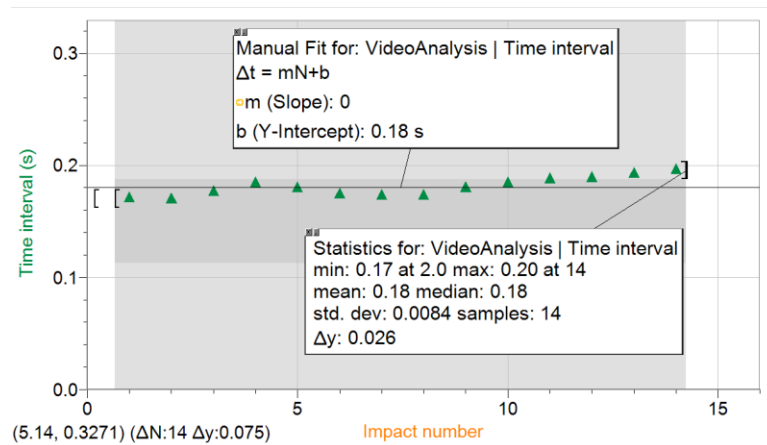


**Fig 2** – wave plot in Audacity.

Screen-capture the image from a wave plot in Audacity and import it to Logger pro. (QuickTime must be installed on your computer).

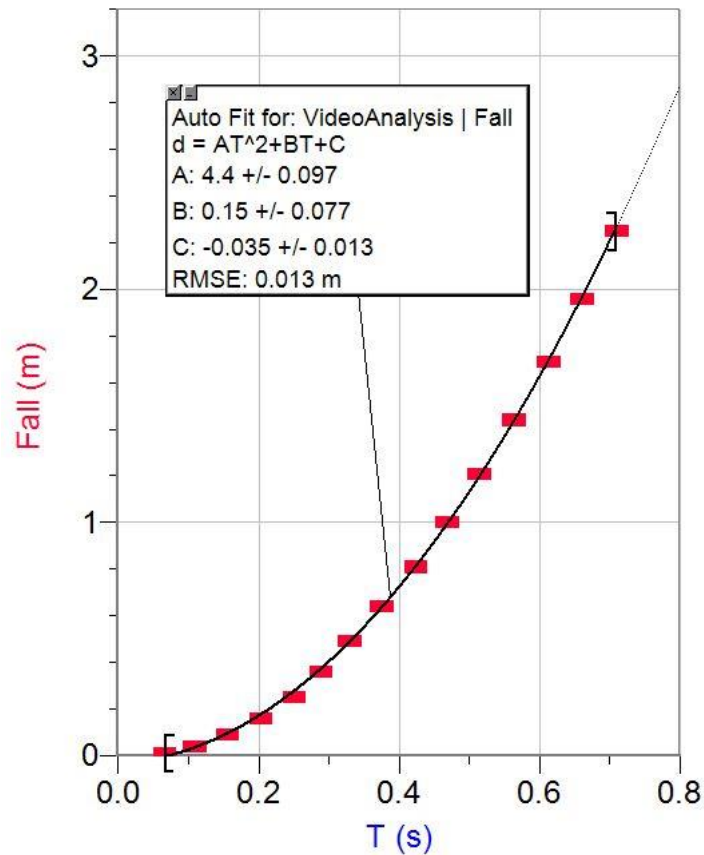
## Analysis

We know the separation of the washers on the thread and we can measure the time intervals between impacts in Audacity. Graph 1 shows the measured time intervals, which are the same neglecting small random errors.



**Graph 1** - time intervals measured from the Audacity plot.

Graph 2 is a distance time plot in Logger pro.



**Graph 2** – distance –time plot for the falling thread.

As expected for constant acceleration the points are a good fit to a parabola.

$$s = ut + \frac{1}{2}at^2$$

The constant A is expected to be  $9.8/2$ . The measured value  $(8.8 \pm 0.2)/2$  is low, probably because of air resistance.

### Questions:

- What is the tension in the uppermost segment of string as the line is falling?
- The thread was light. What would be the effect of replacing the thread by a metal chain?
- If one washer (say number 10 from the bottom) was replaced by a group of five washers what differences (if any) might be heard as the line fell onto the metal tray?