

# The ideal gas equation $PV = mRT$

## Demonstrations

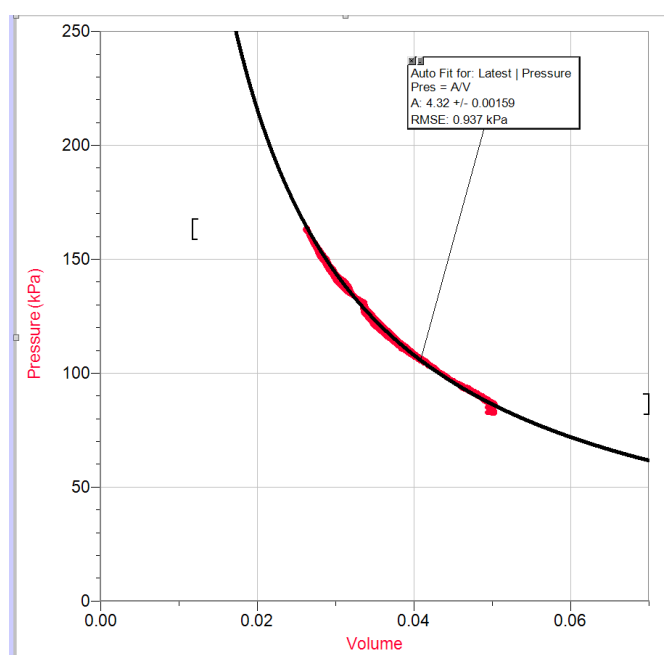
### 1 Pressure versus volume at constant temperature

A plastic syringe is firmly fixed to the bench on wooden blocks with double-sided tape. A Vernier pressure sensor is connected to the syringe. A motion detector plots the position of the plunger in real time.



Note: a low-friction glass syringe could be used for this demonstration but a cheap plastic syringe is satisfactory. A 5 or 10 ml narrow-bore syringe allows the plunger to be moved by hand without excessive force.

A calculated column in Logger Pro gives volume data with zero volume at infinite pressure. Plotting the PV graph in real time as the plunger is slowly moved in and out by hand shows data points moving up and down a preset inverse function line.

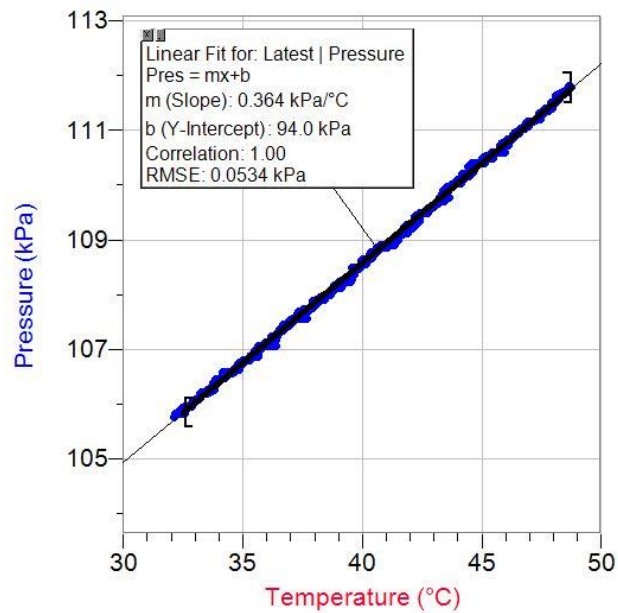


Pressure is inversely proportional to volume at constant temperature.

## 2 Pressure versus temperature at constant volume

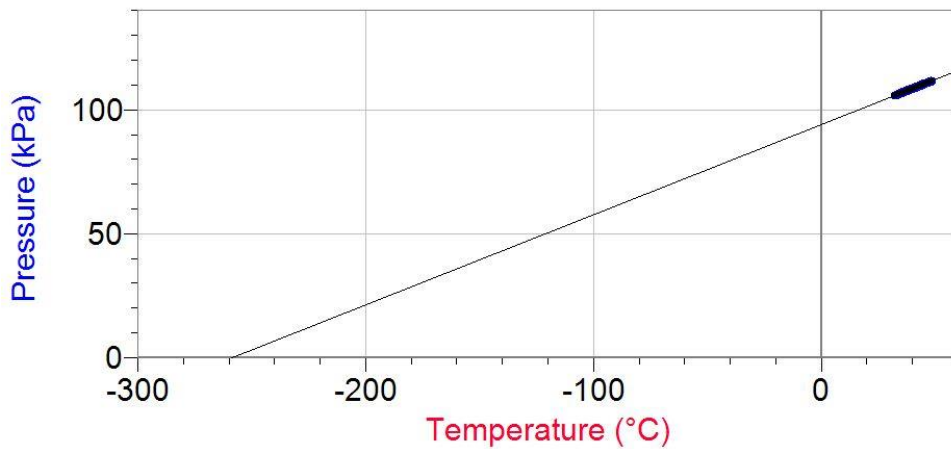


A PET bottle filled with air is connected to a pressure sensor and submerged in water in an extended pot. The water is warmed and constantly stirred.



As the temperature rises the pressure/temperature plot is a straight line.

Extending the pressure/temperature graph to include an origin at zero pressure gives an intercept on the temperature axis at ~ minus 260 degrees Celsius.



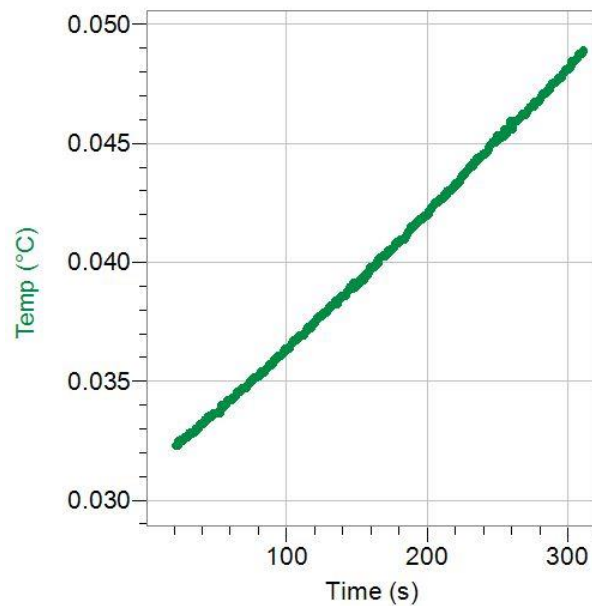
The intercept suggests an absolute zero of temperature at about -260 °C. Pressure is then proportional to the absolute temperature. An extrapolation of this extent is not normally justified but this initial estimate of absolute zero is close to the known value of -273 °C. *It is important when doing this demonstration to eliminate leaks from the bottle. Failure to do so results in a false estimate of absolute zero that is much closer to 0°C.*

The line fit on the Pressure/Temperature graph above gives the pressure as ...

$$P = 0.364T + 94.0$$

... where  $T$  is in degrees Celsius.

Rearranging for  $T$  and entering a new calculated column in Logger Pro allows the water temperature to be plotted as a function of time.



The bottle is a simple model of a constant volume gas thermometer.

### 3 Pressure versus the enclosed mass of air at constant volume and temperature

A large coke bottle with a volume of about 3 litres is fitted with a tire valve and can be pressurized to 4+ atmospheres with a foot pump. The cap is sealed with aquarium sealant. The internal gauge pressure can be measured with a digital pressure gauge.

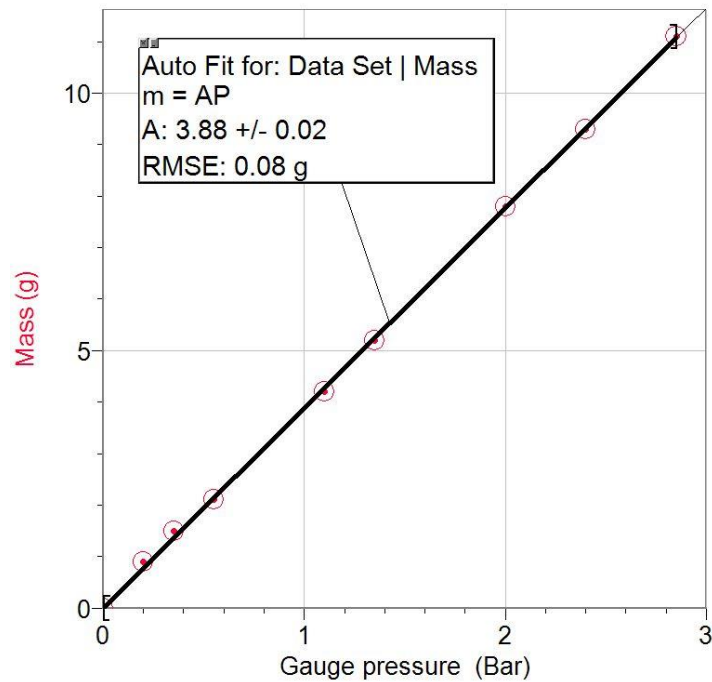


Expanded polystyrene balls inside the bottle (mostly trapped air) act as a pressure gauge to add visual appeal of the demonstration.

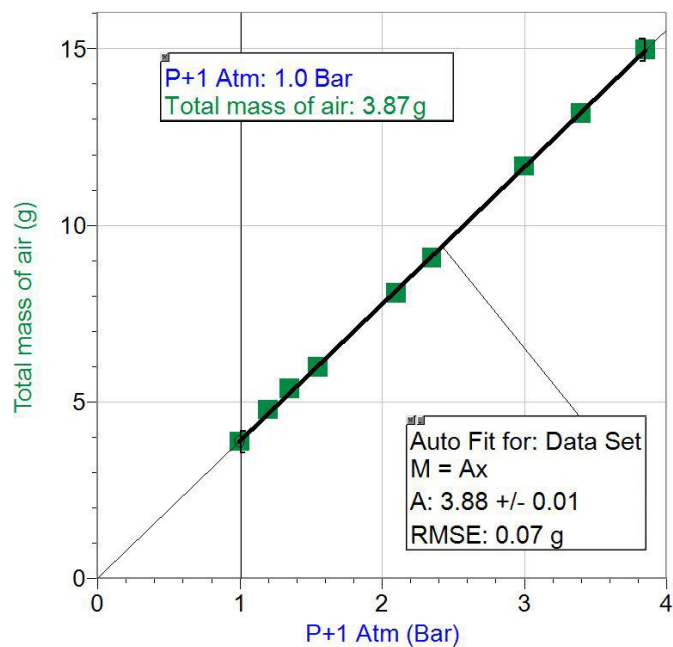


Gauge pressure: zero above and ~3 Atm. below.

Releasing air in steps and plotting the excess mass against the pressure gives the straight line plot below.



Extending the straight line to an origin at zero pressure converts gauge pressure to absolute pressure.



Pressure is proportional to the mass of air in the bottle and the density of air is confirmed as  $\sim 1.3 \text{ kg per m}^3$ , accurate enough for demonstration purposes.

## The ideal gas equation

The relationships demonstrated above are ...

$$P \propto 1/V$$

$$P \propto T$$

$$P \propto m$$

... where  $P$  is the pressure in Pascals,  $V$  is the volume in cubic metres,  $T$  is the absolute temperature, and  $m$  is the mass of enclosed air in kg.

Combing the three equations and transposing gives the pressure relationship as ...

$$PV \propto mT$$

Inserting a constant (for air) gives ...

$$PV = mRT$$

... where  $R$  is the *specific gas constant*.

*Specific gas constants* in J/(kg K) are measured for different gasses and listed in tables.  $PV = mRT$  is the original form of the ideal gas law that can be used to find the lift of a hot air balloon or the mass of air contained in a room.

### The universal gas constant

If the equation is rewritten as  $PV = nRT$  where  $n$  is the number of moles of gas enclosed (directly related to the mass  $m$ ) then the gas constant takes a value that is the same for all gasses at low pressures. The *universal* gas constant  $R$  is 8.21 J/(mol.K).

Note: specific heats are listed in J/kg. *Molar specific heats* are not the same for different substances and their use has no advantage.

### Additional comments

An ideal gas is imagined to have elastic atoms that occupy no volume with no Van Der Waals forces between atoms. Air at atmospheric pressure is assumed to behave like an ideal gas. *The same symbol  $R$  often indicates either the universal gas constant or a specific gas constant. Be careful to identify which one is meant.* Demonstration 1 is done over several seconds per cycle to avoid adiabatic heating and cooling. The constant volume gas thermometer is restricted to  $<70^\circ\text{C}$  using PET plastic. It could be improved a little by shortening the hose connecting the bottle to the pressure sensor and by compensating for the thermal expansion of the bottle. A demonstration of Charles' law,  $V \propto T$  at constant pressure, with a mercury capillary tube returns a poor estimate of absolute zero. This method is preferred.