Helmholtz Resonance

An enclosed volume of air is connected to the atmosphere by a short pipe.

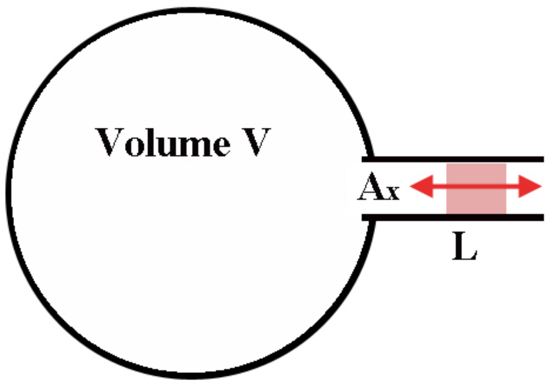


Fig 1 - the dimensions of an idealized Helmholtz Resonator

If air oscillates in the pipe the pressure is raised and lowered in the enclosed volume *V*. If the restoring force due to pressure fluctuations is proportional to the displacement of the mass of air in the pipe we will have simple harmonic motion. Showing that, and writing down an expression for the frequency of such an oscillation requires careful thought. Pressure change ΔP in the enclosure is not simply proportional to the change in volume ΔV, because compressions are adiabatic and the temperature rises and falls with the oscillation. Adiabatic compression must also be accounted for in the derivation of an expression for the speed of sound in air.

The math

For adiabatic compressions it can be shown that PV^{γ} is constant, where γ is the ratio of specific heats c_P/c_V … (approximately 1.4 for air). Using this result without proof …

$$\Delta P/P = -\gamma \Delta V/V \qquad \ldots [1]$$

… when the wavelength of the sound is much greater than the dimensions of the enclosure and the pressure change ΔP inside at any instant can be taken as the same everywhere.

The reduction in volume of the cross sectional area of the pipe A_x times the displacement of the oscillating air Δx. The restoring force is given by …

$$f = \Delta P\, A_x$$

$$= \gamma\, (\Delta P/V)\, (A_x)^2\, \Delta x$$

The mass of air in the pipe is $\rho A_x.L$ where ρ is the density of air at atmospheric pressure, A_x is the cross sectional area of the pipe and L is the effective length. L is a little longer than the pipe because air oscillates outside each end.

Writing the equation of motion … $d^2x/dt^2 = - f/m$ … in terms of f and m gives …

$$d^2x/dt^2 = - [\gamma \Delta P/V) (A_x)^2 \Delta x]/\rho A_x L$$

$$= - [\gamma P A_x/\rho V L] \Delta x$$

The motion is simple harmonic with frequency given by …

$$f = 1/2\pi [\gamma P A_x/\rho V L]^{1/2}$$

The speed of sound in air (derived in a similar way) is given by …

$$v = [\gamma P/\rho]^{1/2} \qquad \ldots [2]$$

… and the natural frequency of oscillation becomes …

$$f = v/2\pi [A_x/VL]^{1/2} \qquad \ldots [3]$$

The oscillation is referred to as Helmholtz Resonance after Professor Helmholtz, the German academic who first described the effect in the 18th century.

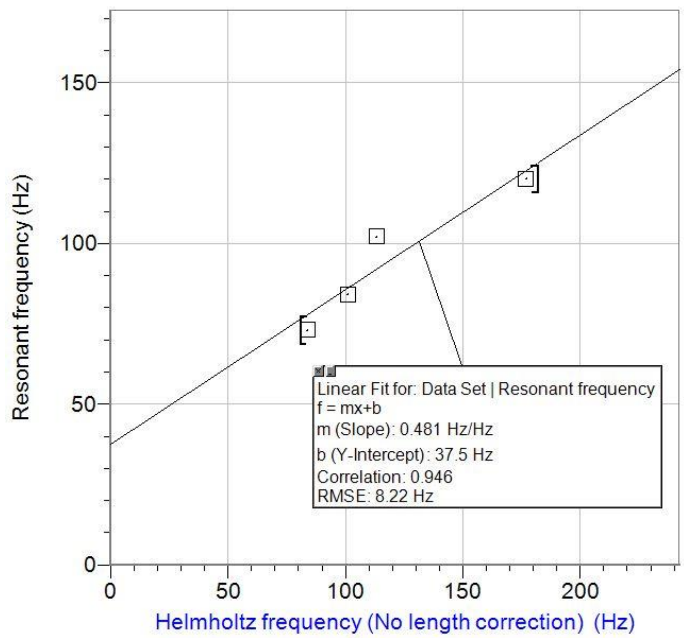
Demonstration

The café-style coffee container in figure 2 has a volume of 775 cm^3. This one is of a sturdy construction. Flimsy examples do not resonate as well.

Fig 2 - the components of a demonstration Helmholtz resonator.

The round hole in the cap (figure 2) has a diameter of 1.45 cm. Short lengths of plastic pipe with an internal radius of 0.65 cm convert the container into a Helmholtz resonator.

A pipe is inserted a short way (2 mm) into the cap. Placing a bottom lip on the top of the pipe and blowing gently excites resonance that drops in frequency as the pipe is lengthened. Plotting the measured resonant frequency against a calculated frequency using equation 3 gives the linear plot below.



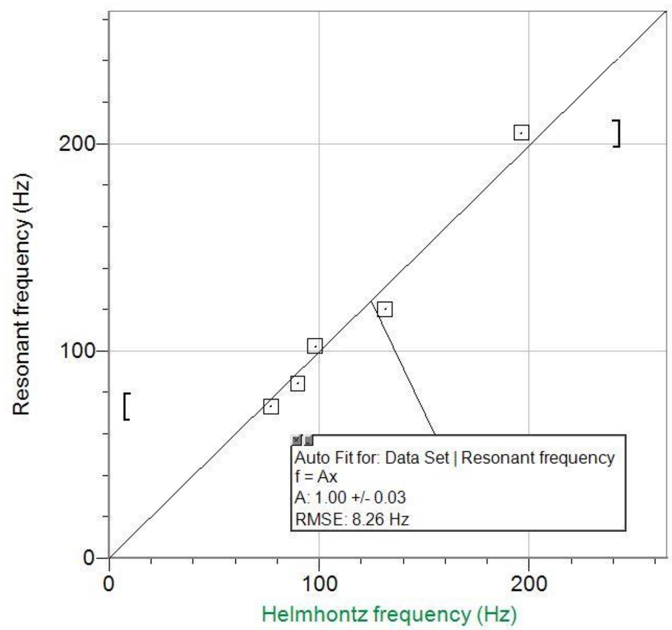
Graph 1 - the data does not closely conform to the calculations.

The slope of this graph is less than unity and line of best-fit passes nowhere close to the origin, showing that the Helmholtz relationship [equation 3] does not give the measured frequencies.

The length correction

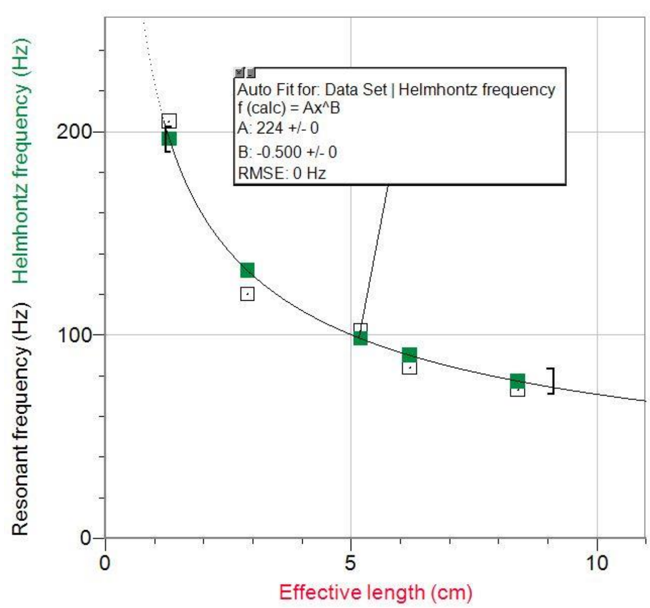
There is movement of air outside the ends of the pipe, which increases the mass of the vibrating column so it is reasonable to replace the actual length of the pipe with an effective length. The gradient of Graph 1 becomes 1.0 when 1.3 cm is added to each of the pipe lengths (including zero). This length correction was found by repeated calculation (trial and error) using the expression below in Logger Pro.

f = (34000/6.28)*((3.14*0.65^2)/(("Neck length"+"Length correction")*775))^0.5



Graph 2 - with length correction of 1.3 cm.

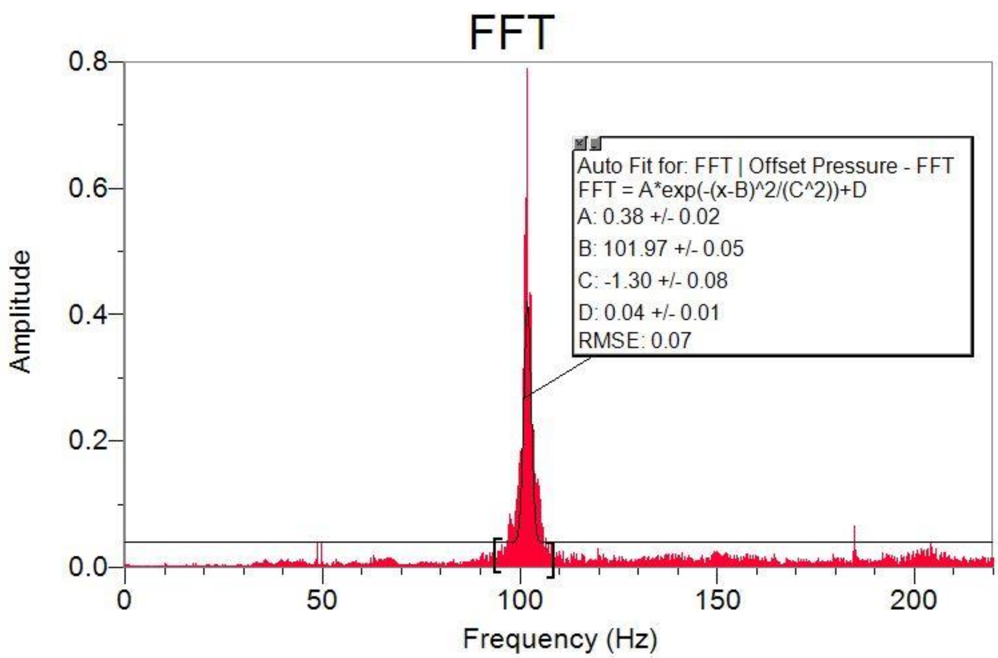
The combined length correction, (the sum of inner and outer end corrections), is of the order of magnitude of the pipe diameter. In this case the outer end is baffled by the lips and mouth and the correction depends somewhat on the velocity of the airflow. ***When demonstrating:be as consistent as possible and blow gently***. Plotting measured and calculated frequencies against the effective pipe length, (*L*+1.3) cm, gives a square root plot.



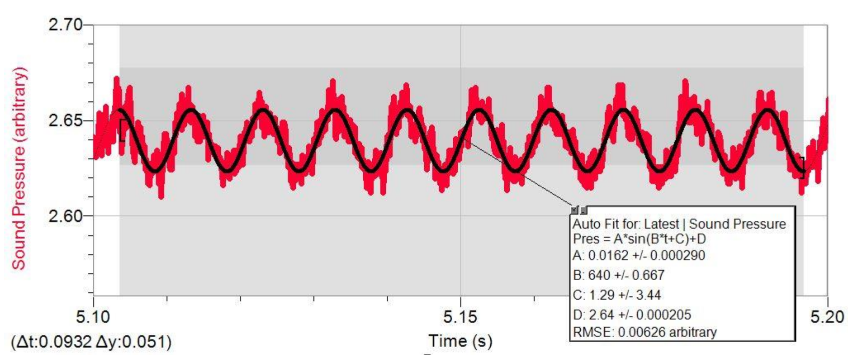
Graph 3 – measured and calculated frequencies with length correction.

Appendix

A representative FFT spectrum with a Gaussian function fit to the resonance peak is shown below when the pipe length was 3.0 cm.



Since only the fundamental is present the frequency can equally well be determined as $\omega/2\pi$ by fitting a sine function of the wave plot.



Values of frequency obtained by either method agree to within one Hz. Slightly larger variations are due to the method of excitation. If a signal generator is available the arrangement could be excited to resonance with a frequency sweep but for demonstration purposes it is often better to take the personal approach.