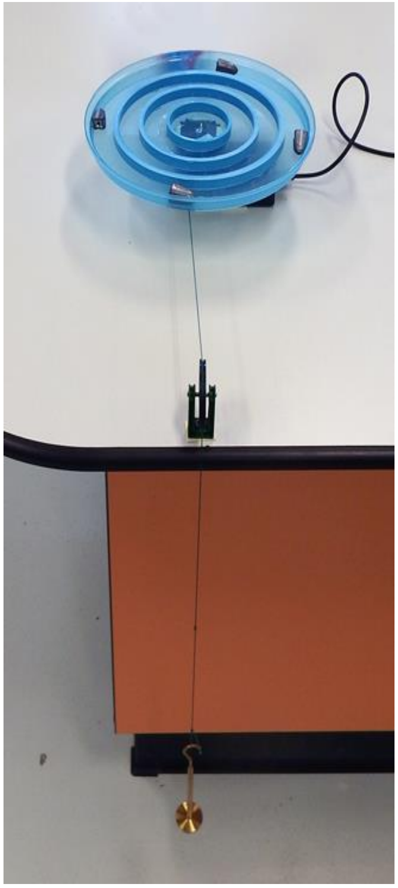
Questions: moments of inertia

Question 1

A plate with concentric rings is mounted on an angular motion detector. A string that is wound in a single layer on a spool of 1.45 cm radius passes over a free running pulley to a falling mass of 30 grams. Four small masses have been added to the plate to increase the moment of inertia.



Part 1

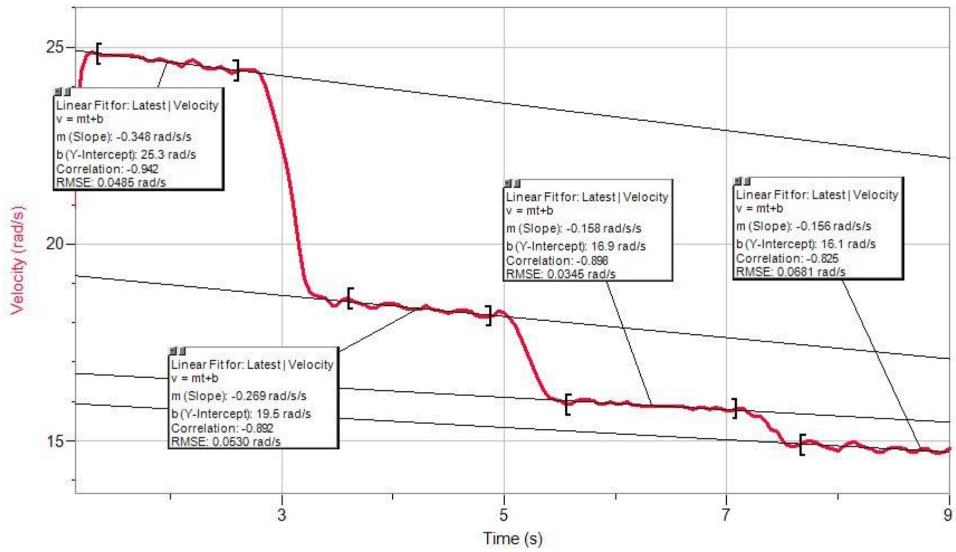
a How could the apparatus be used to find the moment of inertia of the empty plate? What measurements and calculations are involved?

b How could the apparatus be used to show that the moment of inertia of a point mass *m* is mr^2 where *r* is the distance from the rotation axis?

c How could the apparatus be used to show that energy is conserved during the descent of the falling mass.

Question 1: Part II

The string is removed and the plate is set spinning. The angular momentum is to be found before and after mass is dropped onto the plate. *The instructor suggested that three 30 g masses be dropped at the same time. A student heard that but decided to drop one mass at a time into the outer ring because that was easier*.



a Why do you think the fitted straight lines have negative slopes and why do the slopes decrease following each mass drop?

b Why would you expect to see the same negative slope for all similar lines on a velocity/time graph made by dropping mass in this way into a box sliding across a level floor?

c Why do you think the changes in velocity took place over a third of a second and were not instantaneous?

d The student read angular velocity before and after each collision and calculated the angular momenta. He found that his results had errors of ±10%. *How might he improve this accuracy*?

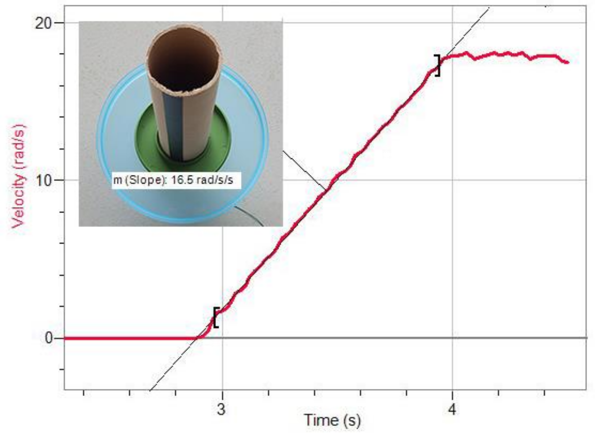
Question 1: Part III

a Write down an expression for the total energy of the system in terms of I_0, m, ω, θ (the angle turned through) and r (the radius of the spool).

b What graph would you suggest the student plot in Logger pro to verify the conservation of energy relationship with a straight line, and write down the expected slope of this graph in terms of the known quantities.

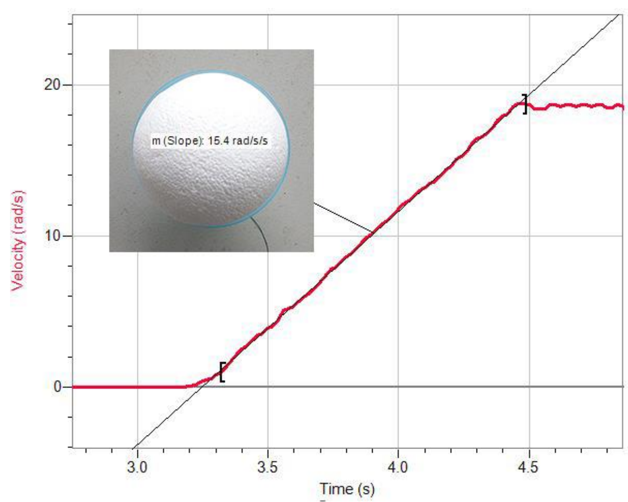
Question 2: Part I

A student was asked to find the mass of a uniform pipe with the apparatus shown in Question 1 above. He put the pipe on the plate, applied a torque and measured the angular acceleration. His angular velocity/time plot is shown below.



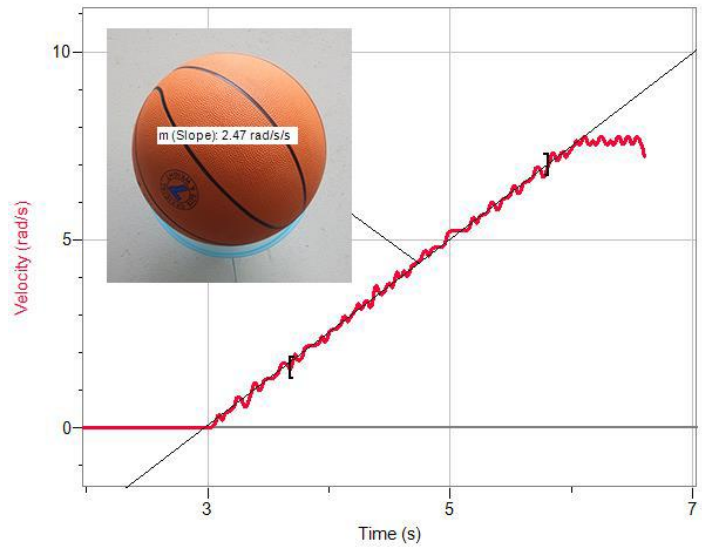
a Find the mass of the uniform pipe, if the torque applied was 0.0165 N m, the pipe diameter was 5.0 cm, the angular acceleration was 16.5 rad/s/s the moment of inertia of the pipe about the axis of rotation is mr^2, and the moment of inertia of the empty plate is $1.20\text{x}10^{-3}$ kg m^{2}.

A second student was asked to find the mass of a uniform solid ball with the same apparatus. She fixed the ball carefully in centre of the plate, applied a torque, and found the acceleration. Her angular velocity/time plot is shown below.



b Find the mass of the ball. The diameter of the ball was 20.0 cm, the torque was 0.0165 N m, the angular acceleration was 15.4 rad/s/s, the moment of inertia of the plate was $1.20\text{x}10^{-3}$ kg m^{2}, and the moment of inertia of a solid uniform sphere about an axis through the centre is $^{2}/_{5}\, mr^2$.

A third student was asked to find the diameter of a basketball. He fixed he ball on the plate and measured the acceleration with an applied torque of 0.0165 Nm. His angular velocity/time plot is shown below.



c Find the diameter of the ball. The mass of the ball was 345 g, the angular acceleration was 2.47 rad/s/s, the moment of inertia of the empty plate was $1.20\text{x}10^{-3}$ kg m^{2}., and the moment of inertia of a hollow uniform sphere about an axis through the centre is $^{2}/_{3}\, mr^{2}$.

Question 2: Part II

The density of air at normal atmospheric pressure is 1.2 kg/m^3.

a Find the mass of air in the ball at 1.5 atmospheres pressure.

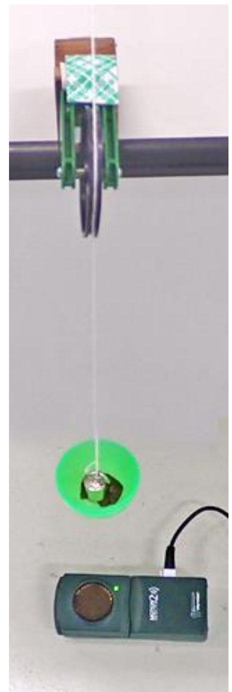
b Why would this extra mass have almost no effect on the moment of inertia as measured?

When answering Question **a** in **Part I** above you were expected to assume that the cylinder was thin-walled.

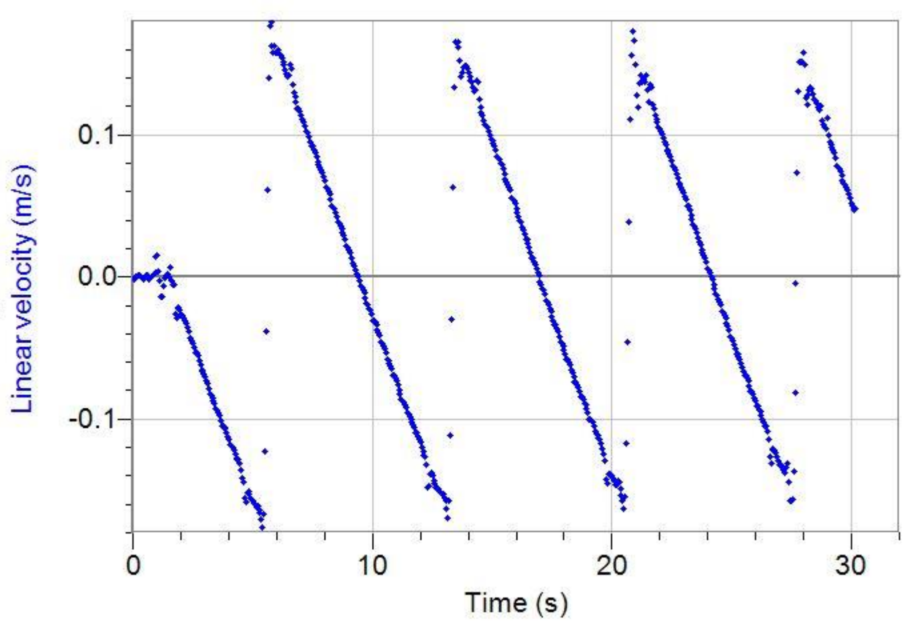
c If the inner diameter was 4.8 cm and the wall thickness was 4.0 mm. estimate the error in your answer to **a** in **Part 1** due to neglecting the wall thickness.

Question 3

A student set up the apparatus shown in question 1 above and put a motion detector on the floor as shown.

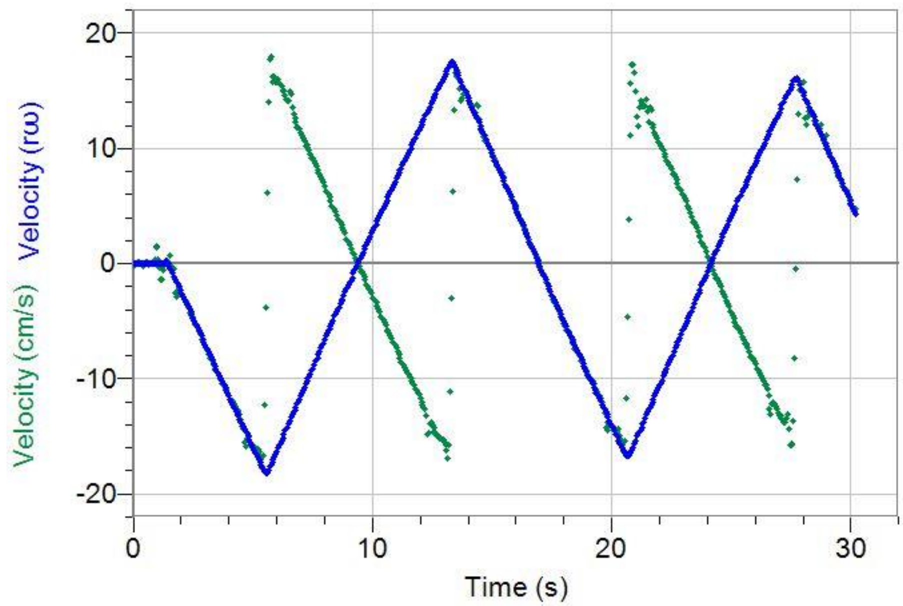


He plotted the vertical velocity *v* against time.



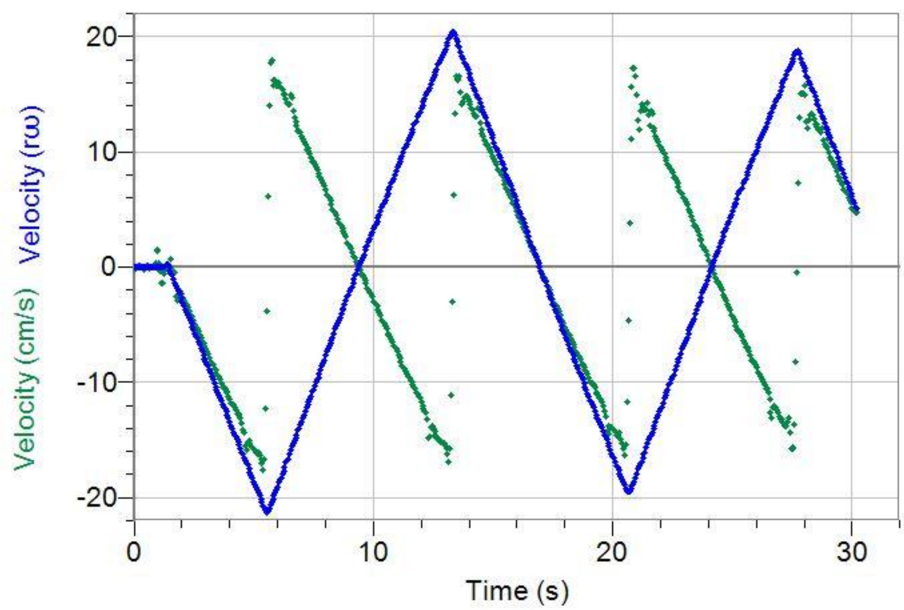
a Why, in physical terms, is this graph a series of straight lines?

He then plotted the vertical velocity, v, and the quantity $r\omega$ against time on the same graph.



b Interpret this graph. Explain clearly what is happening between 2 and 6 seconds and between 6 and 13 seconds.

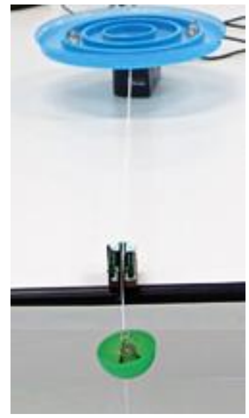
His friend plotted a similar graph but found that lines did not match up properly.



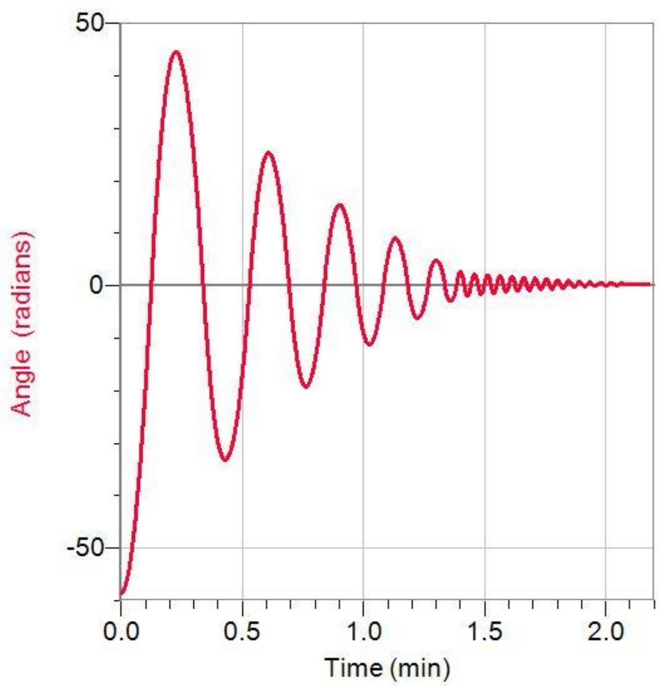
c Give advice. What quantity has probably been measured incorrectly?

Question 4

A circular disc is mounted on a Vernier angular-motion detector and accelerated by a torque provided by a hanging weight.



String is wound on a spool under the disc and the mass is released. The system oscillates.

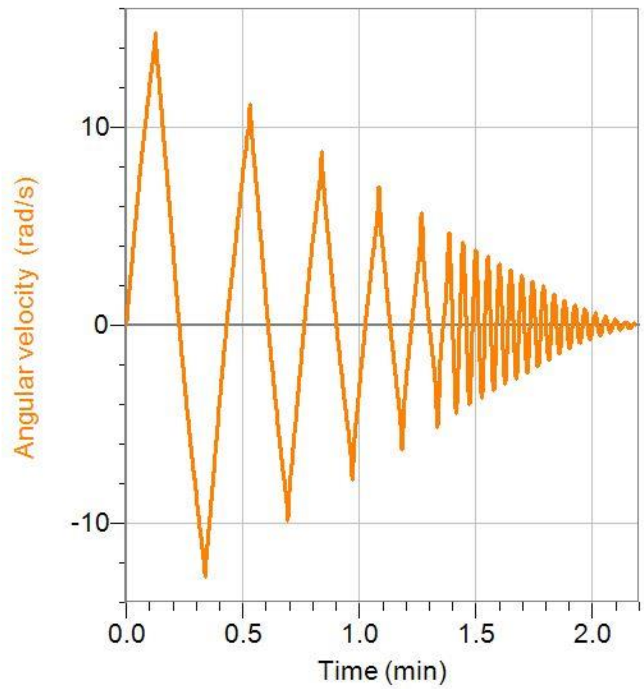


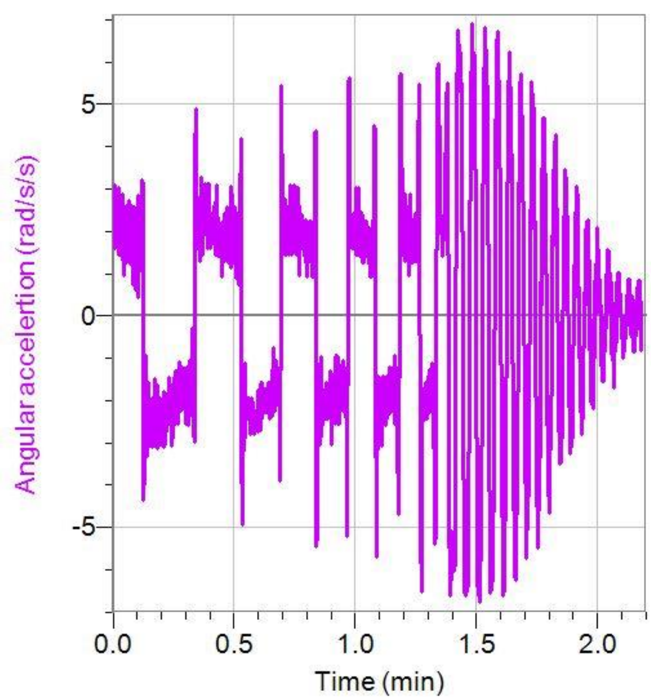
a Explain simply how you know at once that the oscillation is not simple harmonic.

b Describe (by inspecting the graph) how the oscillation changes abruptly in both period and damping at 1.3 min.

c What do you think might be the reason for this abrupt change?

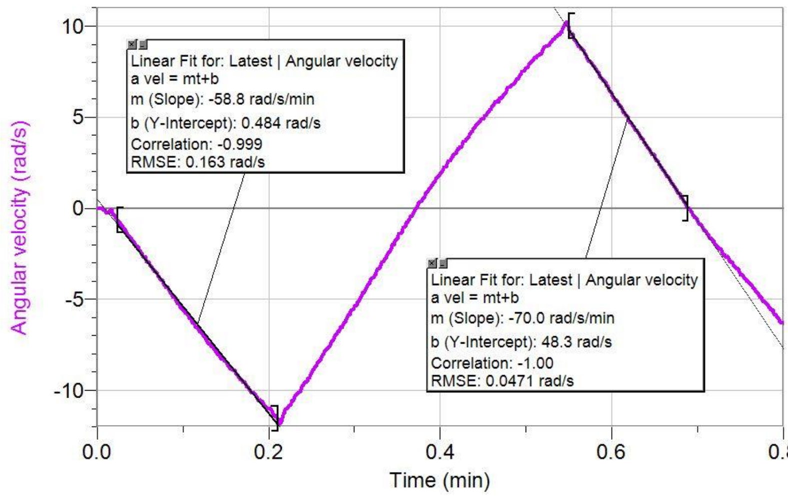
Corresponding angular velocity and angular acceleration graphs are shown below.





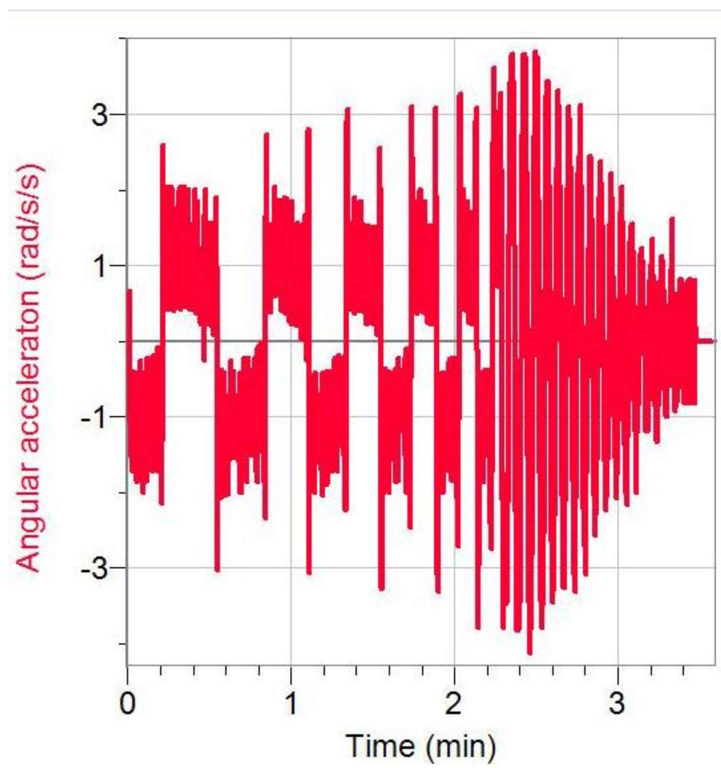
b Look carefully at these two graphs and describe the motion in as much detail as you can.

A detail from the angular velocity/time graph above is shown below.



c The graph is a series of straight line sections with different slopes. What is the most likely reason for the difference in slope of each section?

Mass is added to the plate. The new acceleration graph is shown below.



d Outline at least two differences you see in the acceleration graphs and account for them in physical terms?

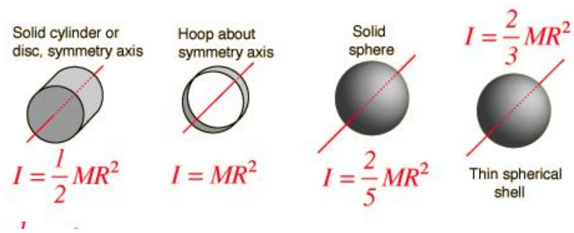
Question 5

When objects roll from rest on an incline without slipping … $mgh = ½mv^2 + ½I\omega^2$

a By making substitutions for moments of inertia and angular velocity show that the final linear velocity squared is given by …

$$v^2 = 2gh/(1+k)$$

… where k is the constant in the moment of inertia relationship kmr^2.

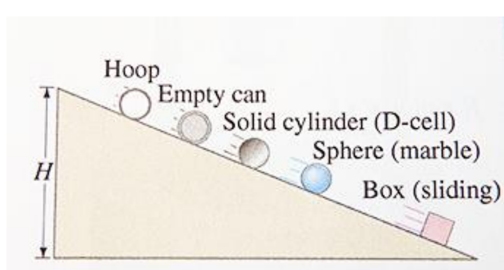


b Explain simply why all uniform solid spheres rolling without slipping on an incline have the same velocities at all times if they began to roll from rest at the same time from the same height.

c Three identical glass soda bottles are empty, filled with water, and filled with cooked rice respectively. All three bottles roll without slipping on a fixed incline from the same height. Place the bottles in order of increasing velocity (slowest first) with brief explanations.

d A reel is made from a smooth hollow pipe and two discs. Place the following in order of increasing velocity (slowest first) if they roll without slipping from rest from the same height on an incline: the reel, the pipe on its own, and the disc on its own.

f This figure from a text shows the order of descent of uniform objects released from rest from the same height at the same time. The diagram could be misleading. Explain why.



Question 6

A soccer ball of radius *r* and mass *m* is hung on a light string of length *r* from a rigid support. The ball is displaced to the side by 30 degrees as shown

a Complete the diagram by marking the position of the center of mass and adding the weight of the ball *mg*.

b Find an expression for the torque acting about the pivot and show that in this simple case the torque is equal to -*mgr*.

c The ball is released. Find an expression for the initial angular acceleration of the ball if the moment of inertia about the pivot is $^{14}/_{3}\, mr^2$ (four and two thirds) mr^2.

d Why (briefly) is the angular acceleration of the ball about the pivot not constant as the ball swings?

e The period of the swing is given by the expression …

$T^2 = 4\pi^2 \, (I_x/mgl)$ .... where I_x is the moment of inertia about the pivot and l is the distance from the pivot to the center of mass.

i Explain (without detailed calculations) why the period of swing about the pivot shown would be less if the ball was a solid uniform sphere of mass *m* and radius *r*.

ii By using I_x (given in part c) and the parallel axis theorem find the ratio of periods for the ball swinging with small amplitude about an axis through X and about a parallel axis touching the surface of the ball.