Planck’s Black Body Relationship

As so often happens in science the first hint of new understanding, in this case the quantization of the energy of a harmonic oscillator, happened as an aside, almost an accident. At the time it went largely unnoticed by the scientific community and was not thought important even by its originator. Max Planck had been trying to derive an expression for the intensity versus frequency relationship for a black body radiator (a cavity with walls in thermal equilibrium). In an effort to rescue his initial failed attempt (in his own words “*ïn desperation*”) he resorted to statistical mechanics. As a mathematical approximation he imagined the oscillators to have discreet energies separated by a small equal energy element that we shall call ε. At the time he thought his artifice had no basis in reality but was merely an aid to calculation. Many physics texts would have us believe in hindsight that he put forward a series of astonishing (unbelievable) postulates but the historical record does not support that contention.

What he did, when others had failed, was to correctly deduce an expression for the intensity relationship at all wavelengths that matched the experimental data of the time. His expression for the power per unit area has a relatively simple form.

$$E_f = \frac{8\pi h f^3}{c^2} \frac{1}{e^{hf/kT} - 1}$$

Derivations in modern terms, making the assumptions about the allowed energy levels of the harmonic oscillators that we now know to be required, are to be found in standard texts and on the web. In 1900 Planck did not have the benefit of this approach and had to resort to his own devices. That he deduced the right expression and found that he could not allow his imagined energy gaps to approach zero, as he had expected, but had to fit the data by making the value of his now famous constant *h* equal to 6.63×10^{-34} Js is truly remarkable.

Demonstrations

Any source of thermal radiation in a school will have a temperature of less than 2500 K and, like a tungsten filament, will be at best only approximately a back body. Peak emission will be between 1200-2500 nm in the infrared, well beyond the range of simple spectrometers and the sensitivity of the eye. The demonstration described below using only a visible light spectrometer and a filament bulb involves unreasonable extrapolation but has value in introducing students to Planck’s function and requires them to think about the difficulties.

In figure 1 a 200 Watt tungsten filament bulb is mounted on a clamp stand close to the end of an optical fibre that is connected to a Vernier visible-region spectrometer. The filament current (and temperature) can be varied with a resistive load.

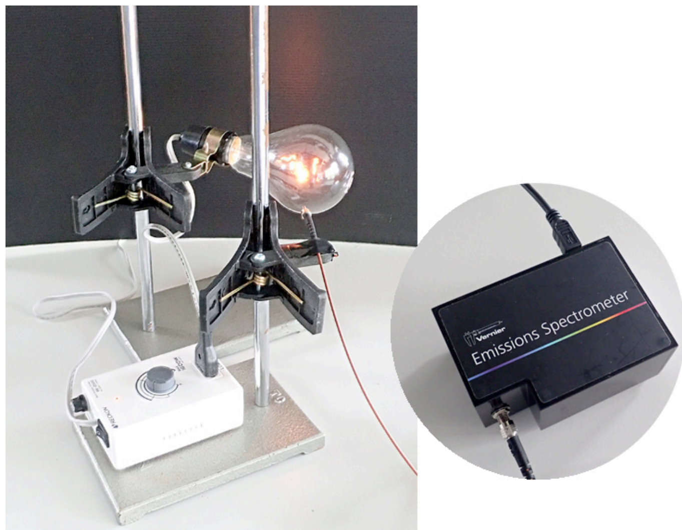


Fig 1 - a resistive load in series with a tungsten filament. Inset: spectrometer.

The filament is made to glow bright orange (at around 1000°C) and the output of the visible light spectrometer is examined.

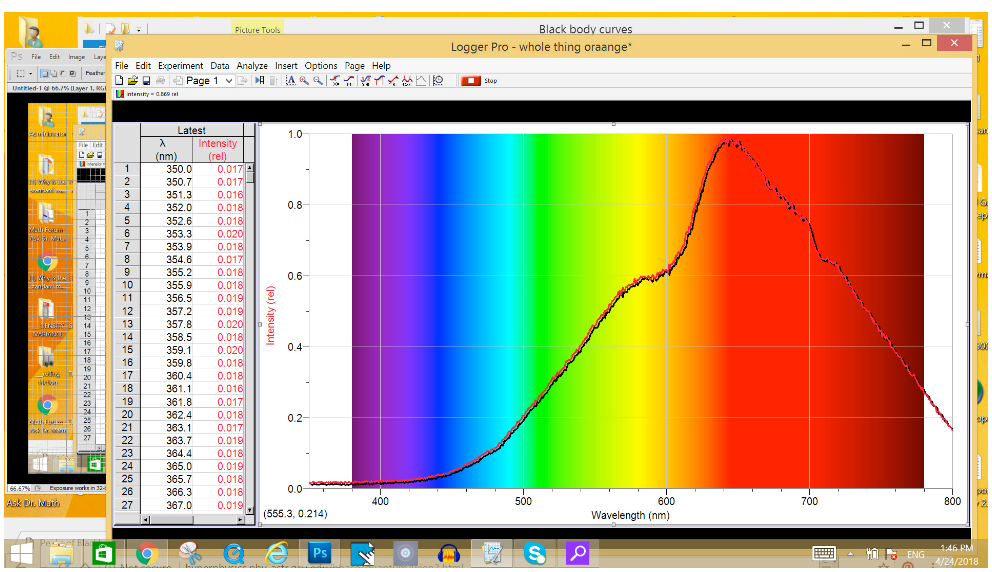
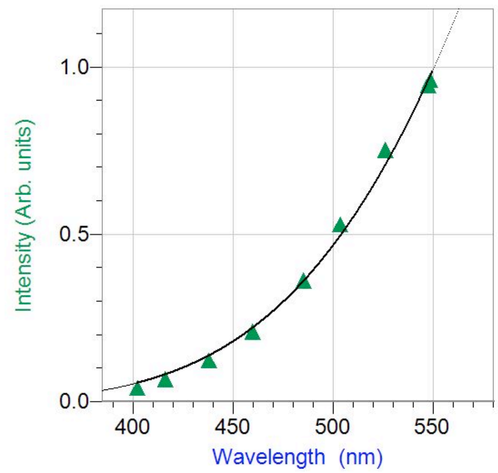


Fig 2 - the spectrometer output from a filament at ~1000°C.

The peak at 650 nm is due to the reducing sensitivity of the spectrometer in the near infrared and has nothing to do with the source. This device cannot be used to plot the expected peak intensity but there is another approach.

Note the steady rise in intensity from 400-550 nm. Depending on the uniformity of response of the spectrometer and the closeness of the filament to a black body this region may represent the first small part of a black body curve.

Inserting figure 2 into Logger pro and marking the curve point by point gives figures 3a and b below.



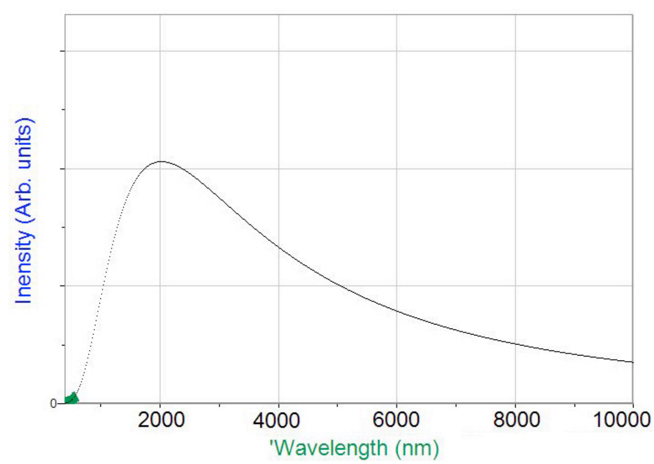
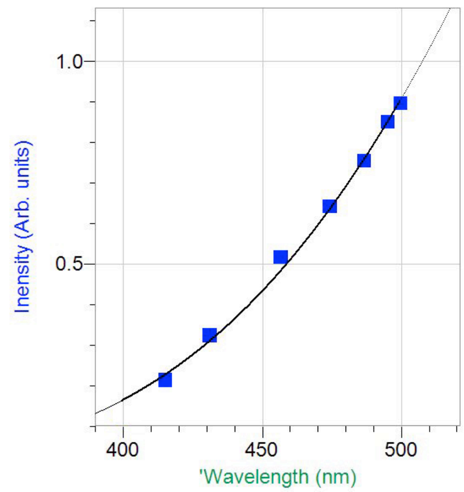


Fig 3a (above) - Planck's relationship fitted to the data points.
Fig 3b (below) – a scale change showing the estimated back body curve.

The relationship in the form (A/λ^3)/(exp(B/λ)-1) has been fitted to the data. The estimated intensity curve peaks at ~2000 nm giving a black body temperature for the bright orange filament of ~1200 °C. For quick reference there is a calculator here. http://hyperphysics.phy-astr.gsu.edu/hbase/wien.html#c3

Increasing the current to the filament increases the intensity of the light dramatically and it becomes more white than orange. The optical fibre input must be moved at least half a metre from the filament. Repeating the procedure gives figures 4a and b below.



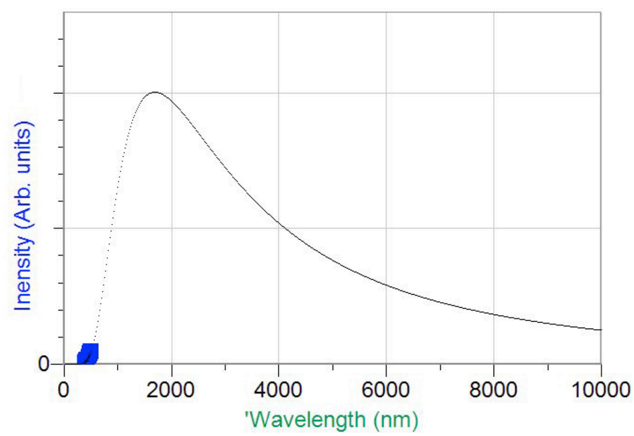


Fig 4a (above) - Planck's relationship fitted to the data points.
Fig 4b (below) – a scale change showing the estimated back body curve.

The intensity verses wavelength curves are crude estimates, but the results do correspond approximately to what is expected. The peak of the second curve is at ~1600 nm and the corresponding black body temperature is ~1500 °C.

Integrating the Planck relationship gives the total radiated power at a given temperature: Stephan's law, $E = \sigma T^4$ for a black body. Stephan deduced his law from experimental data. Note that the input to the optical fibre was moved ~10 times further away when the temperature of the filament was increased. This indicates a very large increase of radiated power with temperature.
A demonstration of the T^4 relationship follows.