

Projectile motion questions

1 A volcanic eruption blasts a rock into the air with an initial velocity of 80.0 m/s at an angle of elevation of 30.0 degrees. (Take g to be -10m/s^2 .)

- Find the time of flight.
- What is the horizontal distance traveled by the rock?
- Determine the velocity of the rock just before impact.
- Calculate the maximum height of the rock above the ground.

(1.)
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2$$

or
$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} + \begin{pmatrix} 0 \\ -10 \end{pmatrix} t$$

The maximum height above launch is $\frac{u_y^2}{2g}$
 $= 80\text{m}$

$$\begin{pmatrix} B \\ -3300 \end{pmatrix} = \begin{pmatrix} 40\sqrt{3} \\ 40 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2$$

or otherwise

$$t^2 - 8t - 600 = 0$$

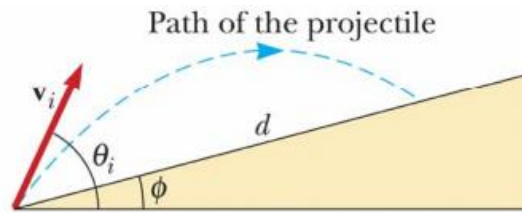
$$(t + 22)(t - 30) = 0$$

$$t = 30\text{s}$$

$$\Rightarrow B = 1200\sqrt{3}\text{ m}$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 40\sqrt{3} \\ -260 \end{pmatrix} \text{ m/s}$$

2 A projectile is fired with an initial velocity v_i at an angle of elevation θ . It lands on an incline (angle ϕ) at a distance d as shown.



Show that the projectile travels a distance d up the incline, where

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g \cos^2 \phi}$$

(2)

Resolving the vector velocity and acceleration along the incline and perpendicular to it...

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u_{ix} \\ u_{iy} \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

$$\begin{pmatrix} d \\ 0 \end{pmatrix} = \begin{pmatrix} v \cos(\theta - \phi) \\ v \sin(\theta - \phi) \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} -g \sin \phi \\ -g \cos \phi \end{pmatrix} t^2$$

... and $t = \frac{2v \sin(\theta - \phi)}{g \cos \phi}$

$$d = \frac{2v^2 \cos(\theta - \phi) \sin(\theta - \phi)}{g \cos \phi} - \frac{g \sin \phi \cdot 4v^2 \sin^2(\theta - \phi)}{2g^2 \cos^2 \phi}$$

$$= \frac{2v^2}{g \cos^2 \phi} \left[\cos \phi \cos(\theta - \phi) - \sin \phi \sin(\theta - \phi) \right] \sin(\theta - \phi)$$

$$= \frac{2v^2 \cos \phi \sin(\theta - \phi)}{g \cos^2 \phi}$$

3 A Hydra base is located at the origin (0, 0). The Hydras plan to launch a projectile to hit a hospital directly to the East with a velocity of 50 km/h at an angle of elevation of 30°.

An Avenger base is located 50 km South and 100 km East of the origin. The Avengers have a canon aimed to the North.

Question: At what initial speed and angle of elevation should the Avengers fire to intercept the Hydras projectile if both projectiles are fired at the same time and $g = 10\text{m/s}^2$?

(3)

At 100 km East and height h ...

$$\begin{pmatrix} 100 \\ h \end{pmatrix} = \begin{pmatrix} 50 \cos 30 \\ 50 \sin 30 \end{pmatrix} t - \frac{1}{2} \begin{pmatrix} 0 \\ -10 \end{pmatrix} t^2$$

$$t = \frac{100 \cdot \sqrt{3}/2}{50}$$

$$= \sqrt{3} \text{ s}$$

$$h = 50 \frac{\sqrt{3}}{2} - 5 \cdot 3 = (25\sqrt{3} - 15) \text{ km}$$

The intercepting shell

$$\begin{pmatrix} 50 \\ h \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} \sqrt{3} + \frac{1}{2} \begin{pmatrix} 0 \\ -10 \end{pmatrix} \cdot 3$$

and $u_x = \frac{50}{\sqrt{3}}$ and $u_y = \left(25 - \frac{15}{\sqrt{3}}\right) + 15 \text{ km/h}$