Projectile motion

Motion with constant acceleration is described by …

$$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$$

… where the variables *s*, *u* and *a* have been written as vectors.

Writing this equation with column vectors gives …

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2 \qquad \text{…. [1]}$$

The column vector equation in t^2 defines the parabolic **trajectory** (path) of a projectile near the surface of the earth when g is taken as constant and the verticals as being parallel. Without these approximations the path is an ellipse and the math is complicated.

As a simple example: set u_x and u_y equal to 30 and 40 m/s respectively and plot the position at intervals of one second. The maximum height is 80 m, the range is 240 m, and the time of flight is 8 seconds.

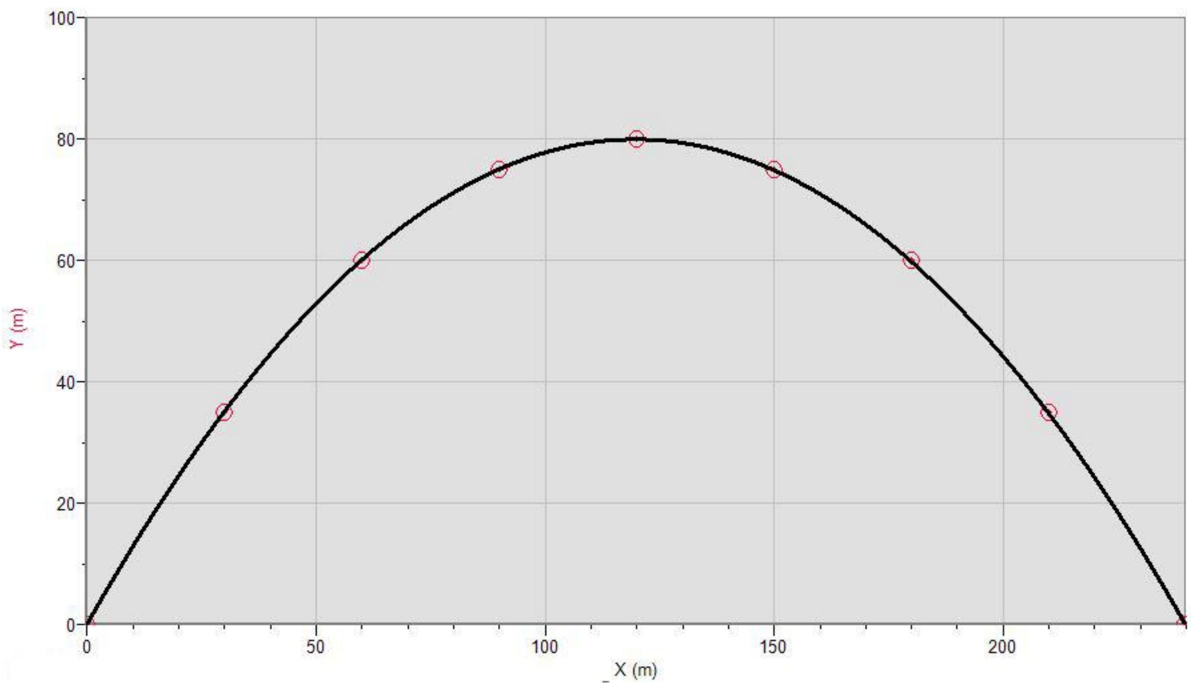


Fig 1 – the parabolic trajectory.

The horizontal velocity is constant (30 m/s) and the vertical acceleration is -10 m/s/s/.

When x is the range R, y is zero.

$$\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix} t + \tfrac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$

The bottom line of the equation gives the time of flight as …

$$t = \frac{2u_y}{g} \qquad \text{… [2]}$$

… substituting t in the top line gives the range as …

$$R = \frac{2u_x u_y}{g} \qquad \text{…[3]}$$

The **maximum height** is given by the value of y when t is half the time of flight …

$$h_{max} = \frac{u_y^2}{2g} \qquad \text{…. [4]}$$

Maximum range for a given initial kinetic energy occurs when the product $u_x u_y$ is a maximum: that is, when the angle of launch is 45°.

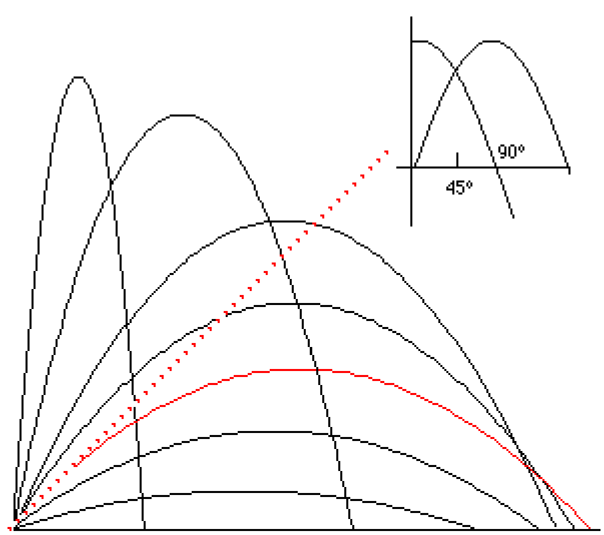
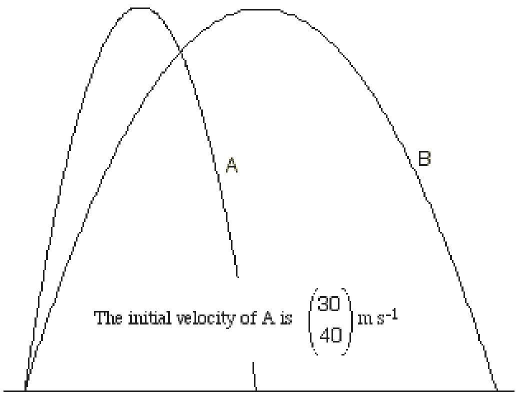


Fig 2 – range versus angle of launch.

Questions

1 Two trajectories are shown with the same maximum heights. The diagram is not to scale but the range of B is twice that of A



i Write down the time of flight of B if A is in the air for 8 seconds.

ii Write down the velocities and accelerations of A and B at the top of their flights as column vectors.

iii Write down the initial and final velocities of A and B as column vectors.

2 If air resistance is not important a stream of water follows a parabolic path. A photograph can be imported to Logger pro with video analysia and the path can be fitted to a parabola.

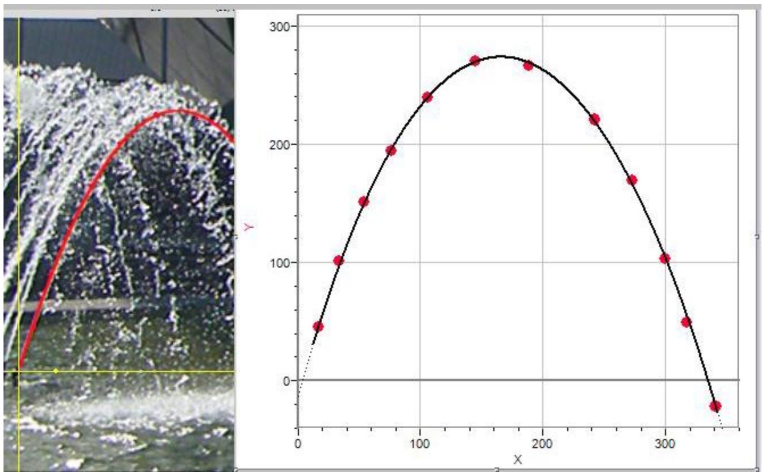


Fig 4 – the parabolic trajectory.

Outline two common examples for which air resistance would be more important than here. Selet examples for which the dominat reason is differnet, Explain your reasoning.

3 One of the conditions of agreeing to work with me is to have some imagination. Suppose we could go to the moon for the afternoon and walk around freely as we do here.

a Write down the equation of the path of a golf ball near the surface of the moon.

[g_{moon} = -1.6 m/s/s]

b Find the range of the golf ball on the Moon if the range on earth is 240 m and the initial conditions are the same.

c Find the time of flight on the moon if the time of flight on earth of 8.0 seconds and the initial conditions are the same.

d Find the maximum height of the ball on the moon if the maximum height on earth was 80 meters and the initial conditions were the same.

e Think about your answers to questions a - d and describe the feeling of hitting and watching the ball on the moon. How would that experience be different from the experience on earth?

4 Moon sports

a How would you expect the human high-jump and long-jump records to be extended for athletes in a pressurized Moon dome?

b Design a moon based golf course. Would you suggest we modify the ball to make the game possible and enjoyable? If so How?

c Design a moon based basketball court and suggest any modifications that might be necessary to make the game payable and interesting to watch.