Ringing Rods and Pipes

Bending modes

Two cylindrical solid aluminium-alloy rods were taken from a set of chime bars and cut to the same length, as shown in figure 1. One rod has a hole close to one end.

Fig 1 – cylindrical aluminium-alloy rods of almost the same length.

The rods ring when dropped vertically on a hardwood parquet floor. Frequency spectra recorded in Logger Pro for 10 drops over 5 seconds are shown below.

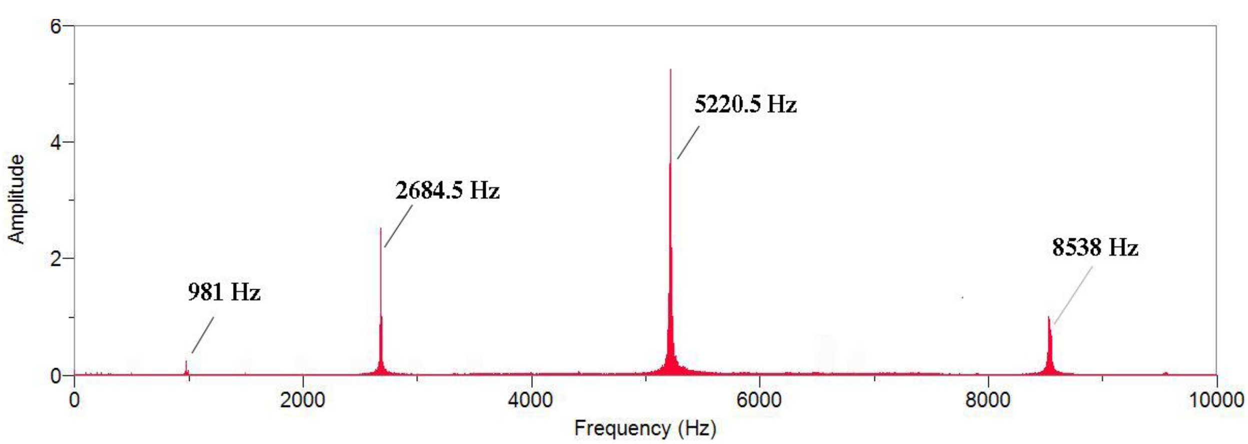


Fig 2 – FFT spectrum for the uniform rod bouncing on a hard parquet floor.

The FFT spectrum for the uniform rod shows a series of transverse bending modes. The separation interval increases as the frequency increases because the velocity of the transverse waves increases with increasing frequency.

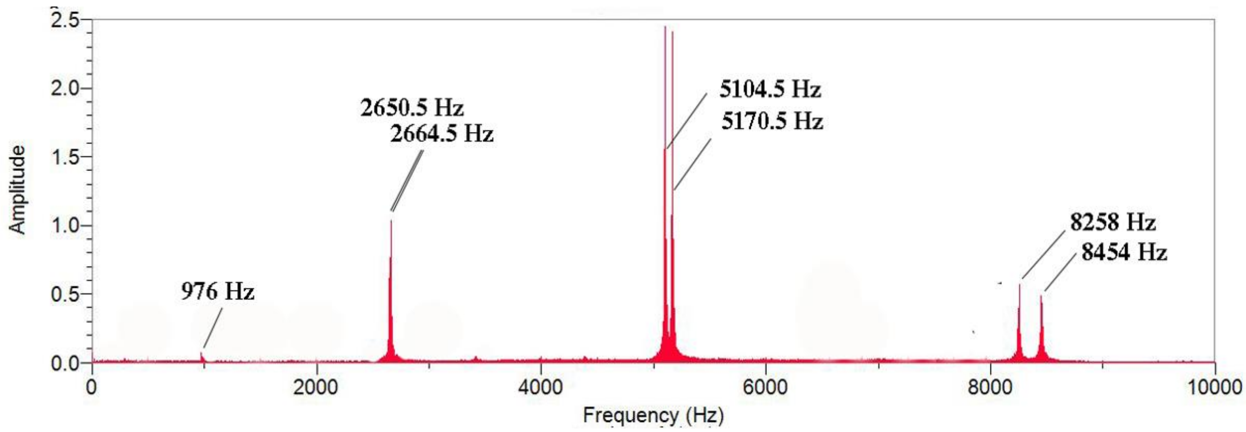


Fig 3 - FFT spectrum for the non-uniform rod with the hole when bouncing on a hard parquet floor.

The FFT spectrum for the rod with the hole shows *two* closely separated series of transverse bending modes at lower frequencies than the series in figure 2.

The bars are close to 160 mm in length and 5.0 mm in diameter. The centre of the hole is 25 mm from the end and 2.5 mm in diameter.

Longitudinal modes

The apparently identical rods came from a single set of wind chimes and are assumed to be of the same material. The speed of sound in aluminium alloys is around 5200 m/s so any longitudinal modes will be above 10000 Hz and not be shown in figures 2 or 3.

The uniform rod was dropped on a ceramic floor tile to excite higher frequency modes. Figure 4 below has been compiled from two FFT plots.

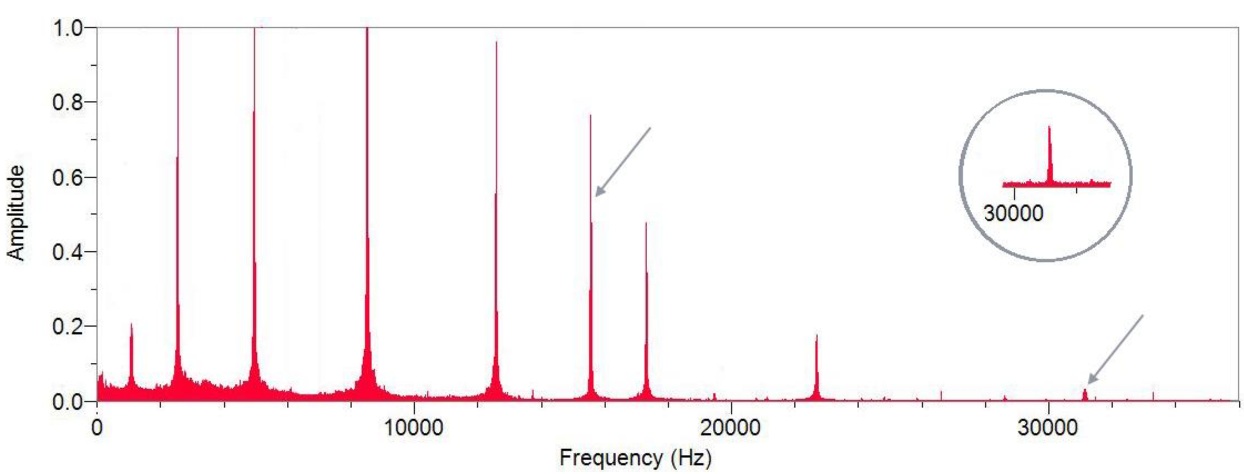


Fig 4 – an extended FFT spectrum for the uniform rod dropped on a tile.

In figure 4 the three bending mode lines below 6000 Hz were copied from a spectrum made by dropping the uniform rod on a parquet floor. Above that frequency the FFT spectrum was recorded with a sampling rate of 71000 per second as the rod was dropped onto a floor tile. To increase the relative amplitude of any longitudinal modes in The FFT plot the microphone was positioned 2 cm above the ringing rod, not at the side as for figures 2 and 3. The bending mode sequence is extended to seven harmonics and two extra lines are identified with arrows. The inset shows the resonance above 31000 Hz with an increased vertical scale.

Extending the frequency scale and marking the centres of the additional lines gives frequencies of 15580 and 31155 Hz. The higher frequency is twice the lower one to four significant figures. The additional resonances are identified as the first non-dispersive longitudinal modes and the speed of sound in the metal is found to be close to 5000 m/s as expected. Some error may be present in this value because of unknown end corrections at the terminal antinodes.

Dropping the rod with the hole on a floor tile shows that the upper bending modes are all split into two components but the longitudinal modes are not.

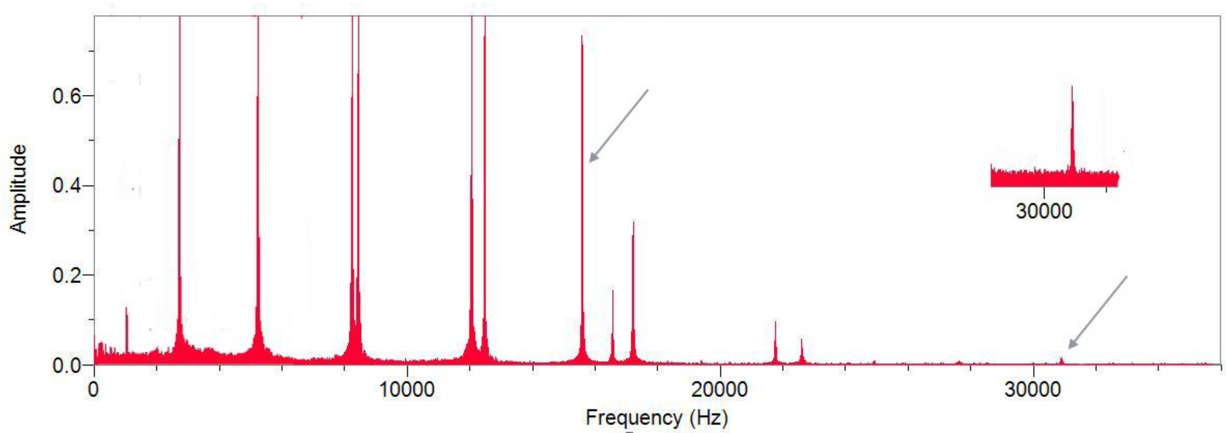


Fig 5 – an extended FFT spectrum for the rod with the hole dropped on a tile.

The longitudinal modes in figure 5 at 15600 and 30905 Hz are of almost the same frequencies as the corresponding modes in figure 4. Close examination shows that the uniform rod was about 0.2% shorter and the cut end is not perfectly flat. The lowering of the frequencies of the bending modes cannot be accounted for on the basis of unequal rod lengths and is due to a different effect.

Questions remain …

1 The speed of sound calculated from the length of the uniform rod and the measured frequencies of the longitudinal modes may be in error due to unknown end corrections. Devise a set of experiments to eliminate this error. *You may assume that you have a grant to purchase what you might need.*

2 The frequencies of the two bending mode sequences and the slight shifts in the longitudinal modes are not simply related by a scale factor. Investigate frequency shifts as a function of hole-size. *You may want to use longer metal bars of larger diameter.*

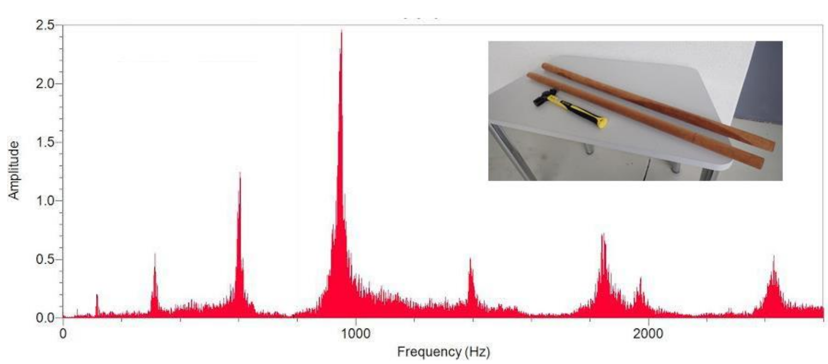
3 All the bending modes above are split progressively by the presence of the hole. Determine the effect of moving a single hole of the same diameter progressively closer to the middle of the bar. Is it possible to position a hole so that one or more of the bending modes is not split?

4 The bending modes are split here into two components. Experiment with other modifications of a bar (two or more holes, and/or notches removed from a bar etc.) in an attempt to split the modes into three or more components. *You may want to use longer metal bars of larger diameter.* If this attempt fails account for that in physical terms?

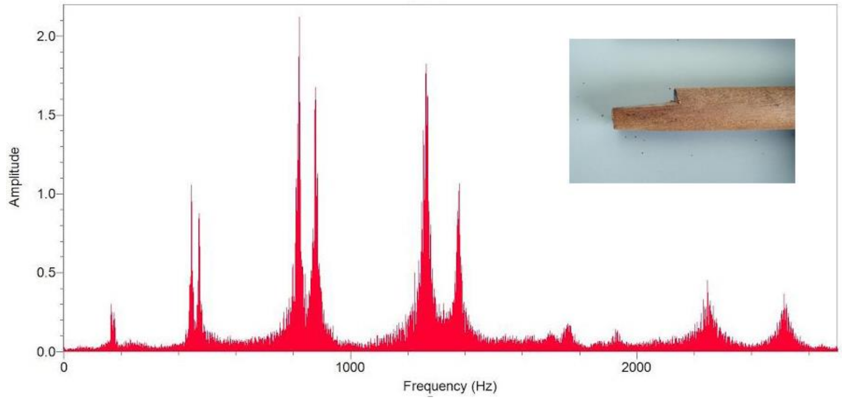
Additional relationships

1 A sudden change in the internal diameter of a hollow pipe is said to lead to partial reflection of sound and the splitting of the longitudinal modes of a vibrating air column. Design apparatus to investigate this effect with two and three partially open or partial closed boundaries.

2 When a slightly irregular one-metre adze handle made from hardwood is dropped vertically on tiles it rings predominantly with a set of bending modes.



Cutting a notch from one end of the bar splits the modes into two components.



Identify the longitudinal mode in the two spectra above and find an approximate value for the speed of sound in the wood.

Note: the bending modes for the modified wooden rod are split into two components (as before) but are moved upwards in frequency, in the opposite direction to the split modes in a metal rod with a hole. Note also that the fundamental is split into two closely spaced components.

Investigate this effect in wooden rods and look for a relationship between the line splitting and the mass of wood removed.

3 Frequency resolution in figure 3 is not sufficient to show any splitting of the fundamental at ~1000 Hz, but splitting of this mode has been observed for metre long aluminium pipes and hardwood rods.

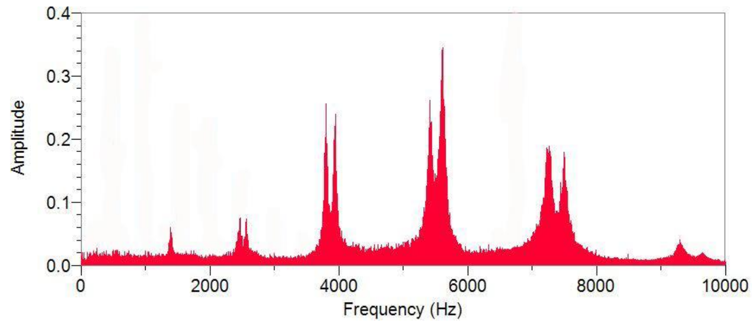
Devise a way to predict the splitting of the fundamental in figure 3 and then plan and conduct an experiment or series of experiments to verify your predicted splitting.

4 The transverse modes on all the spectra shown above have increasing intervals as the frequency increases.

Compare dispersion (the change in velocity with wavelength) for transverse modes on pipes and rods of different materials (metals, woods and plastics).

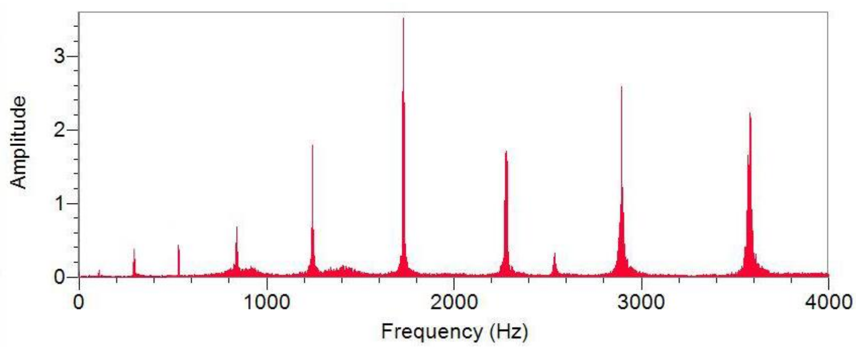
… or investigate a possible relationship between dispersion and stiffness for rods or pipes of the same material.

5 A bamboo chopstick with a square section and a long gently tapered cylindrical point has the frequency spectrum below when dropped on tiles.



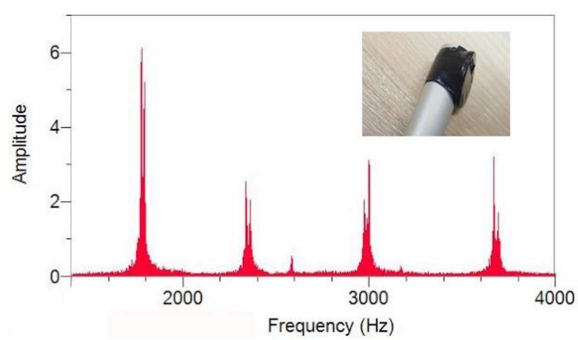
Investigate the mode splitting for chopsticks of different cross sections.

6 A metre long aluminum-alloy pipe is dropped vertically on floor tiles.



Identify the longitudinal mode and find an approximate value for the speed of sound in the metal.

A 10 baht coin is taped firmly to one end of the pipe. Note the slight upward shift of the longitudinal mode in the plot of the split upper bending modes below.



Investigate the effects of adding mass in different amounts at different places. Can locations be found that do not split one or more of the bending modes?