Rotational motion (in three parts)

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Part I: moment of inertia for point masses

As every young man with a motorbike knows, a fat passenger ruins a fast takeoff. Physics types put it this way …

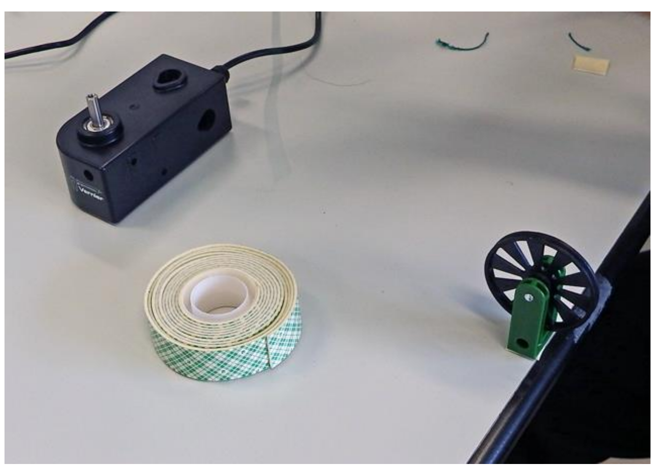
$$f = ma \qquad [1]$$

Angular motion about a fixed axis is described in the same way with force f replaced by torque T (defined as the force multiplied by the perpendicular distance to the center of rotation). Linear acceleration a is replaced by angular acceleration α in radians/s/s. To complete the translation to angular terms an expression must be found to replace mass m in equation 1. The replacement is the resistance of a body to angular acceleration, called *the moment of inertia*. We shall give moment of inertia the symbol I and write equation 1 as …

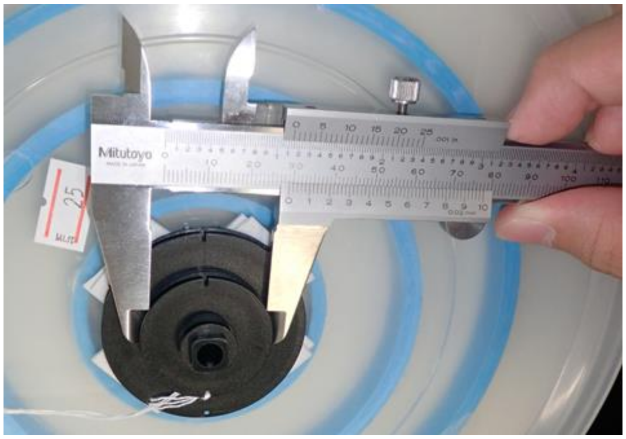
$$T = I\alpha \qquad [2]$$

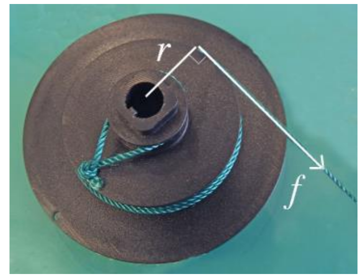
Moment of inertia

An angular motion probe and a light free-running pulley are fixed to a bench with tape.



The pulley and motion detector are mounted so that a string is horizontal and winds naturally onto a spool that has a radius of 1.45 cm.





A plastic plate with rings as shown is fixed to the spool which is then mounted on the angular motion detector.

A falling weight on the end of the string applies a constant torque *fr* to the plate. The acceleration of the plate α is the slope of the angular-velocity/time graph in radians/s/s. From equation 2, the moment of inertia of the plate and rings is given by …

$$I_0 = fr/\alpha \qquad [3]$$

Theory

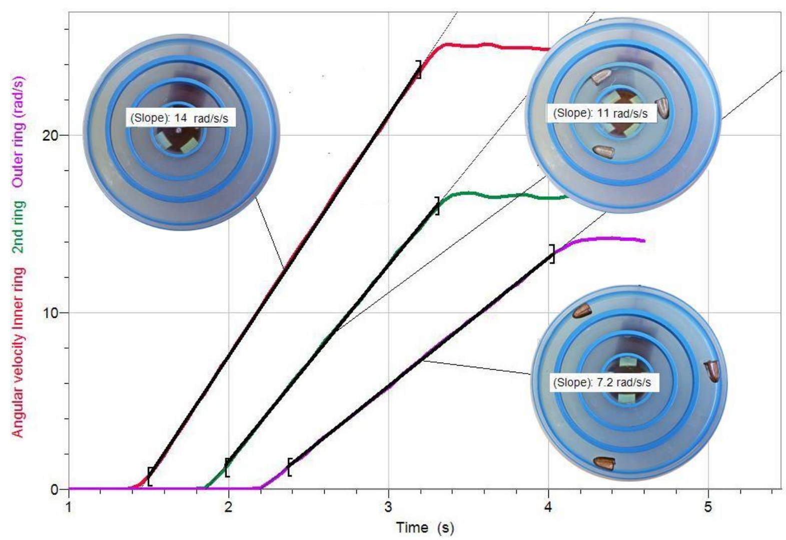
The kinetic energy of a rotating mass is $\frac{1}{2}mv^2$. Since $v = r\omega$, the kinetic energy in angular variables is $\frac{1}{2}[mr^2]\omega^2$. Comparing the two expressions shows that mass m has been replaced by mr^2, defined as the *moment of inertia* of a small (point) mass.

For a collection of point masses, m_1, m_2, m_3, … at radii, r_1, r_2, r_3, … the total moment of inertia is the sum $I_1 + I_2 + I_3 + \ldots$

$$I = \sum_i m_i r_i^2$$

Measurements

The figure below shows angular-velocity/time graphs when the same constant torque is applied to the disc with small masses at different radii.



The figure has been prepared by pasting data columns copied from individual Logger Pro files to *New Manual Columns*. Moments of inertia $I = I_o + I_{3m}$ can be found from equation 3 using the known torque fr and the measured angular acceleration α.

Instructions

Collect data. Find α for different mass distributions with linear fits to angular-velocity/time graphs. Calculate moments of inertia (fr/α) and Σmr^2 in each case. Plot moment of inertia versus Σmr^2. Interpret the graph. Write down I_o. Discuss the significance of a straight line fit to the data points. See [**Analysis**] for details.

Part II: the conservation of angular momentum

For linear motion the net impulse applied to a mass *m* (the force/time integral) is equal to momentum gained $m\,\Delta v$.

For rotational motion about a fixed axis, impulse is the torque/time integral and is equal to the angular momentum gained, $I\,\Delta\omega$. When a mass falls onto a horizontal spinning plate we would expect angular momentum to be conserved in the collision *because there is no external torque*. We would expect to find …

$$I_0\,\omega_0 = I_1\,\omega_1$$

… where the symbols have their usual meanings.

Measurements

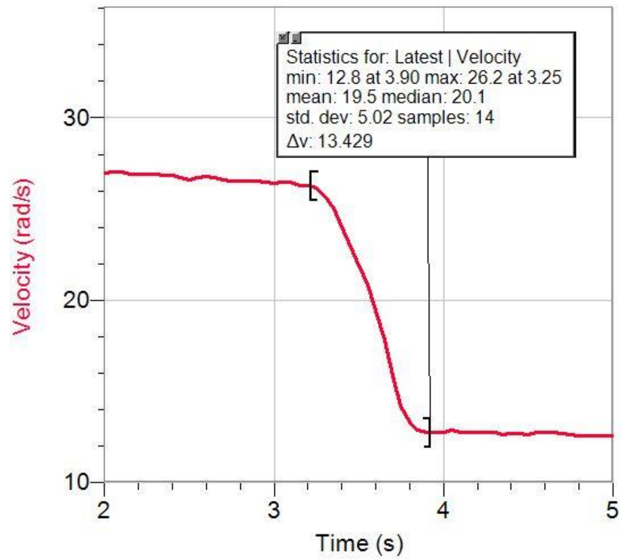
The empty plate is allowed to spin freely on the angular motion detector.

The plate is set spinning. Three 30 gram masses are dropped **at the same time** by hand into the outer ring. The angular velocity of the plate reduces.

> **Note!** If your plate spins too fast masses will be flung across the room and may break computer screens. "*Physicists do it gently: children thrash it.*"

The angular velocity reduction before and after three masses were dropped into the outer ring collision is shown on the graph below. The statistics function has been used to estimate the angular velocities before and after collision.

Angular velocity reduction when three masses are added suddenly to the outer ring.



The moment of inertia was found by calculation to increase from 0.0012 to 0.0023 kg m^2.

$$I_0\omega_0 = 0.0012 \text{ x } 26.2 = 0.031 \text{ kg m}^2 \text{ s}^{-1}$$

$$I_1\omega_1 = 0.0023 \text{ x } 12.8 = 0.029 \text{ kg m}^2 \text{ s}^{-1}$$

$I_0\omega_0 = I_1\omega_1$ are the same within errors: angular momentum is conserved.

Instructions

Collect data and show that angular momentum is conserved when mass is dropped into selected rings.

Part III: the conservation of energy

The falling mass that provides torque in Part I converts potential energy (*mgh*) to both linear and rotational kinetic energy.

Instructions

Either by adding a motion detector to the Logger Pro interface, or by further analyzing your existing files, show that energy is conserved if the effects of friction can be neglected.