The period of a simple pendulum

Introduction

As the name implies, a simple pendulum is the special trivial case of a more common phenomenon, the small oscillations of an object about a fixed point, usually in an essentially uniform gravitational field. For small oscillations (for which sin(θ) can be taken to equal the angle θ in radians) the period is given by…

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

… where l is the distance from the pivot to the centre of mass and I is the moment of inertia about the pivot. [pdf]

When all the mass is concentrated at a fixed distance from the pivot, the distance l becomes the length of the pendulum, the moment of inertia becomes ml^2, and the period relationship reduces to …

$$T = 2\pi\sqrt{\frac{l}{g}}$$

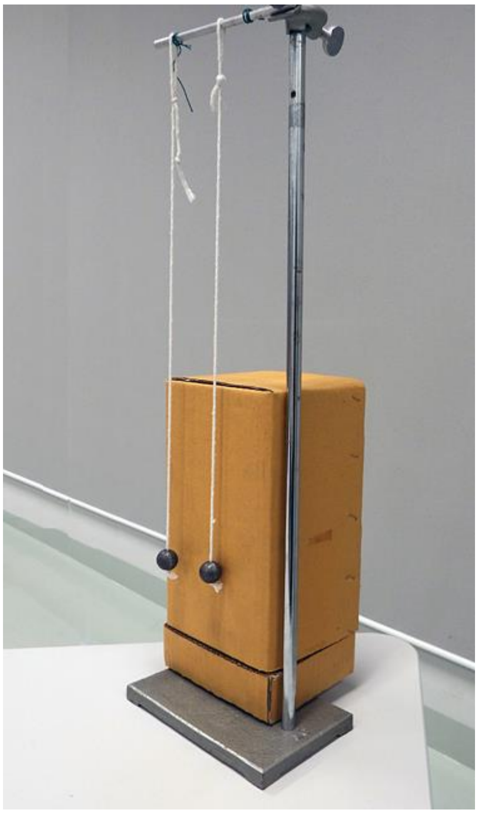


Fig 1 - simple pendulums of the same length.

Amplitude is independent of period

The simple pendulums in figure 1 have the same lengths and will have the same periods. (The loop at the top of the left hand string is secured with a half hitch and the pendulum length can be easily adjusted.)

The balls are held away from the box with the strings at different angles. *Which ball will hit the wall first*? If people watching think about *energy*, *velocity* and *distance* expect hesitant guesses. If they think first about period the answer will be immediate and correct.

The balls will strike at almost the same time if released at the same time.

Almost ... because from initial angles of 20 and 40 degrees the quarter periods are not *exactly* the same. (The small oscillation rule doesn't strictly apply.)

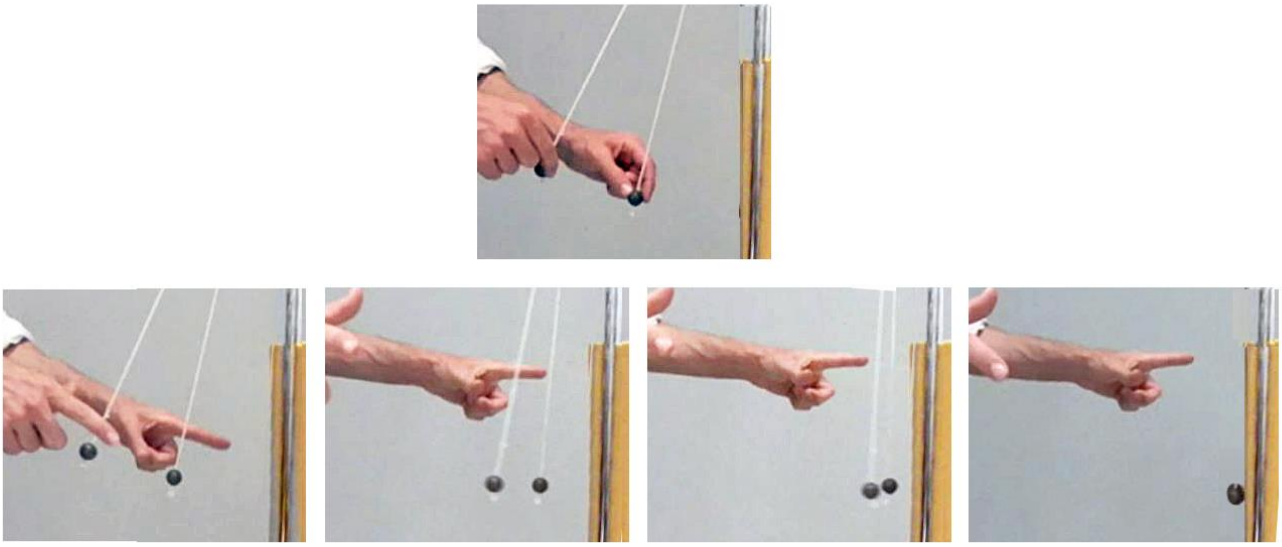


Fig 2 - for convincing demonstration lead balls hit an empty cardboard box with a satisfying single *THWACK!*

Identical lead balls are best for round 1, but the obvious extension to balls of different mass is suggested. Balls of the same radius hit with a single sound as above.

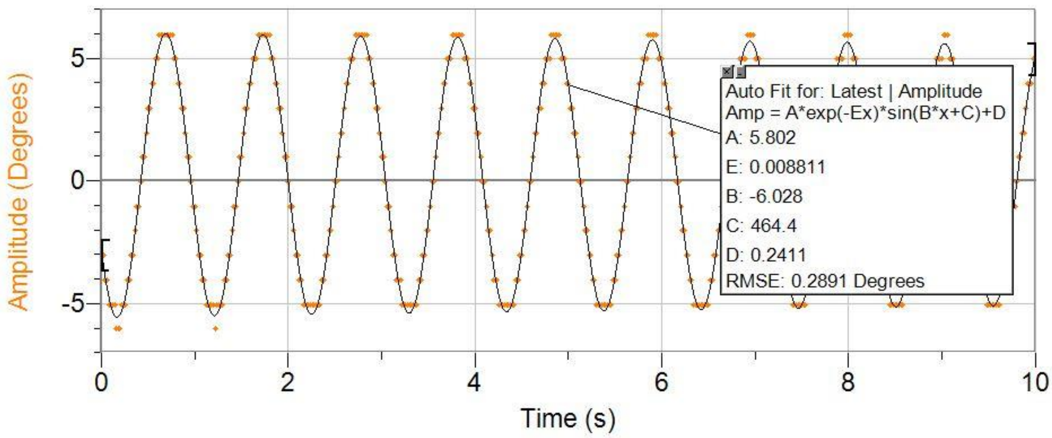
Fig 3 - rubber, lead and wood.

Large amplitudes

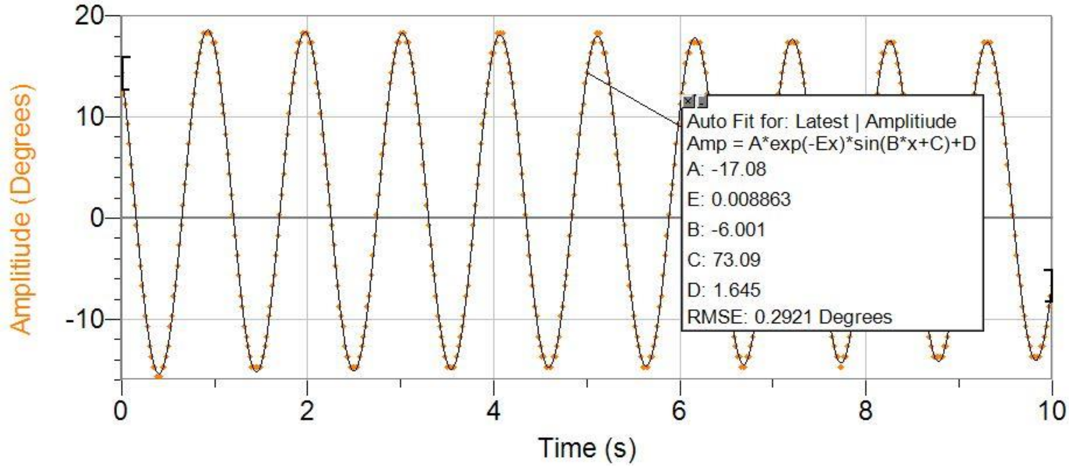
Much is made of the small amplitude approximation. With an angular motion detector demonstration becomes easy and is worth doing.

A simple pendulum is hung on a Vernier angular motion detector**.**

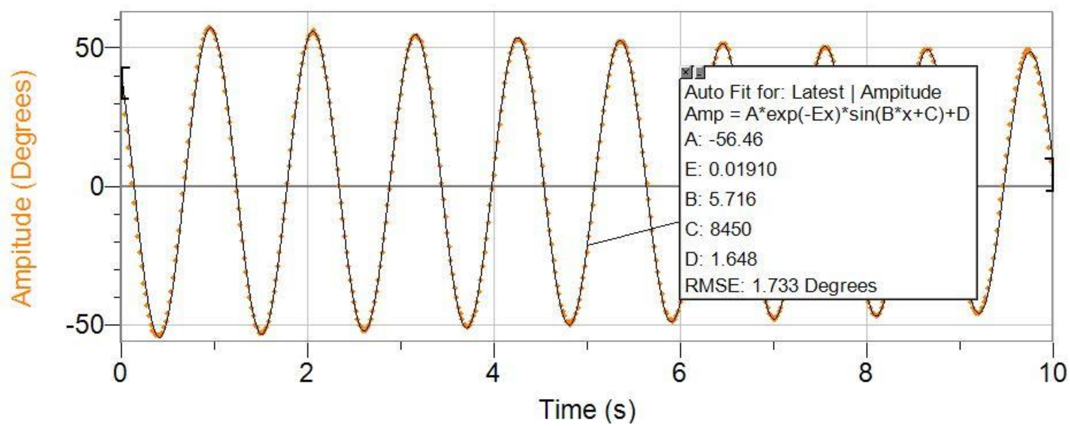
A plot of angle versus time for small oscillations (less than 5°) gives the period as 1.042 seconds from the sine curve fit in Logger Pro.



Graph 1 – small amplitude.



Graph 2 – increased amplitude.



Graph 3 – greatly increased amplitude.

The period, $2\pi/B$, increases slightly with increasing amplitude. The values are 1.042 at 5.5°, 1.047 at 18° and 1.099 at 50°. The period is increased by half a percent at 18° and about five percent at 50°. The values are approximate, for demonstration purposes only. There is a large amplitude period calculator at http://hyperphysics.phy-astr.gsu.edu/hbase/pendl.html#c1

Note: the curve fits above include an exponential decay multiplier.

Exponential decay applies only when the damping force isproportional to velocity. Air resistance (drag proportional to velocity squared) is important here at large amplitudes, and the exponential fit is approximate.