Demonstration: thermal conductivity

Heat is transferred through metals mainly by the diffusion (spreading) of energy in a free electron gas. In non-metals heat is transferred by the diffusion of energy by molecular and lattice vibrations (phonons). With the exception of diamond, resistance to heat transfer is lower in metals than non-metals. Stainless steel is also something of an exception, often described as a poor conductor of heat. Because stainless steel is attractive, hard, and durable, we use it for spoons and food trays: and at KVIS, tumblers for iced water. In these applications a poor conductor of heat is an important advantage. *Stirring hot coffee with a solid silver spoon is an experience better avoided.*

The equation that describes heat transfer by ***thermal conduction*** is …

$$\Delta Q/\Delta t = kA\ (\Delta T/\Delta x)$$

… where the rate of heat transfer, $\Delta Q/\Delta t$ is in J/s, the cross sectional area A is in metres, $(\Delta T/\Delta x)$ the temperature gradient is in K/m. The constant k is defined as the ***thermal conductivity*** of the material. Values of k in J/(m.K) are published in tables (usually at room temperature). The quantity $kA/\Delta x$ is defined as ***thermal conductance***.

> **Note:** the equation has the same form as the electrical conductivity equation $I = C.\Delta V$ with $\Delta Q/\Delta t$ replacing electric current I, and ΔT replacing ΔV. C is defined as ***conductance*** C, the reciprocal of electrical ***resistance*** R. The reciprocal of thermal conductance is *thermal resistance*, given by $R = \Delta x/kA$. The concept of thermal resistance is useful when dealing with heat transfer through extended sheets of layered materials and double glazed windows.

Demonstration

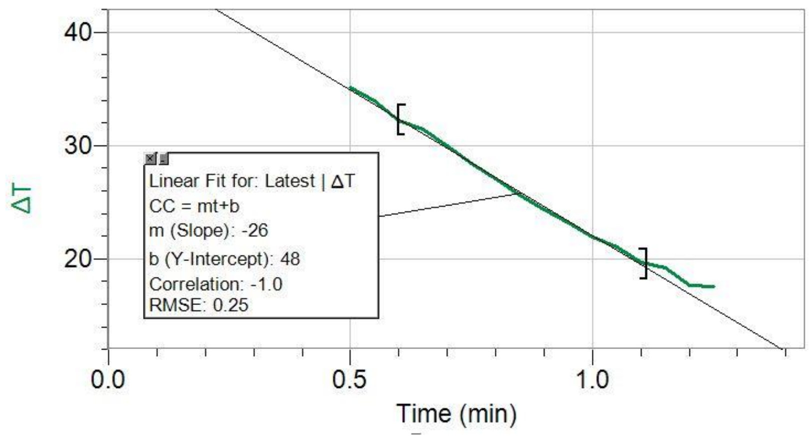
We compare ***thermal conduction*** when a stainless steel pot (k ~16 W/mK) and a polyethylene pot (k ~0.4 W/mK) are placed in hot water.

Fig 1 – plastic, stainless steel, and a water-bath.

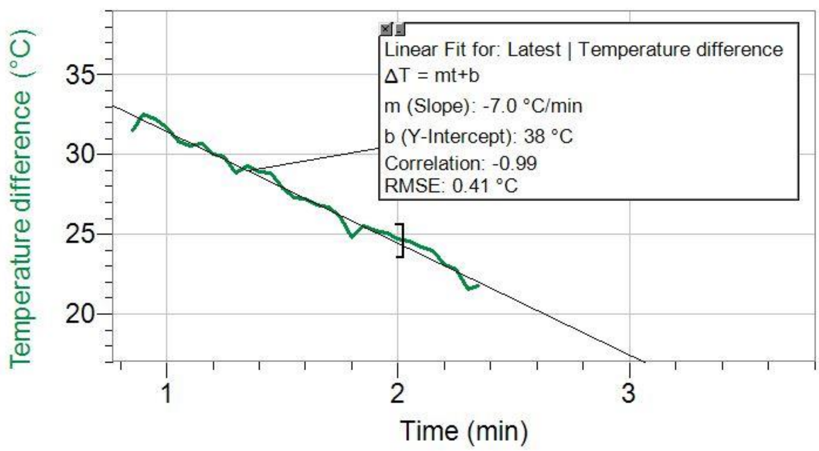
A stainless steel tumbler and a plastic bottle both contain 200 ml of water at room temperature. They are immersed one at a time in hot water, being careful to make the ratio $A/\Delta x$ as similar as possible for each (in this case by submerging the plastic bottle to a greater depth). The total heat Q transferred over a given time is related to the temperature rise ΔT of the cold water by …

$$Q = c_w m_w \Delta T$$

Since the mass of cold water m_w is the same (200 ml) each time the rate of heat transfer per second $\Delta Q/\Delta t$ is proportional to the rate of temperature rise $\Delta T/\Delta t$. Over a small temperature range $\Delta T/\Delta t$ is nearly constant. Temperature-time curves with linear fits are shown below.



Graph 1 – temperature versus time for water in the stainless steel tumbler.



Graph 2 – temperature versus time for water in the plastic bottle.

The temperatures of the water-bath dropped by 1-2 °C per minute. To compensate for this effect graphs 1 and 2 have been plotted as the temperature difference between the bath and the container. Comparing the gradients shows that stainless steel is more conductive but the ratio of heats transferred is about 4:1. A ratio of 40:1 was expected from the published values of thermal conductivities.

The value taken as the thermal conductivity of the polyethylene (0.4) is approximate but the actual value lies somewhere between 0.3 and 0.5. The discrepancy is a factor of 10. **The naïve demonstration has failed**. Why?

The walls are thin and the curvature is low so the heat transfer equation for an extended flat wall will apply to a good approximation. If there is no gross mistake with the rates of heat transfer or the ratio $A/\Delta x$ the mistake is with temperature differences across the materials ΔT_{PE} and ΔT_s.

Towards an explanation

The coupling between phonons (plastic to water) is expected to be relatively good when compared to the coupling of phonons to a free electron gas (water to metal). A water-to-metal interface will involve what is called *an acoustic mismatch.* A water-to-metal interface will add impedance (resistance) to heat transfer that will cause *temperature jumps.* Neglecting temperature jumps for heat transport through windows and walls gives wildly inaccurate estimates of heat transfer. The value of ΔT is not the difference between ambient inside and outside temperatures. Measured surface temperatures of the glass for windows and of the bricks for walls are required to find a value for ΔT.

Taking the published values of thermal conductivities, the estimated heat transfers for the steel and plastic walls, the ratio $A/\Delta x$, and *neglecting the effects of turbulent flow (convection)* gives expected order of magnitude estimates of the actual surface to surface temperature differences as ~20 °C for the plastic and ~2 °C for the steel. This is a preliminary demonstration, not an attempt to obtain accurate values, but the figures show that there is a large interfacial thermal resistance for steel in water.

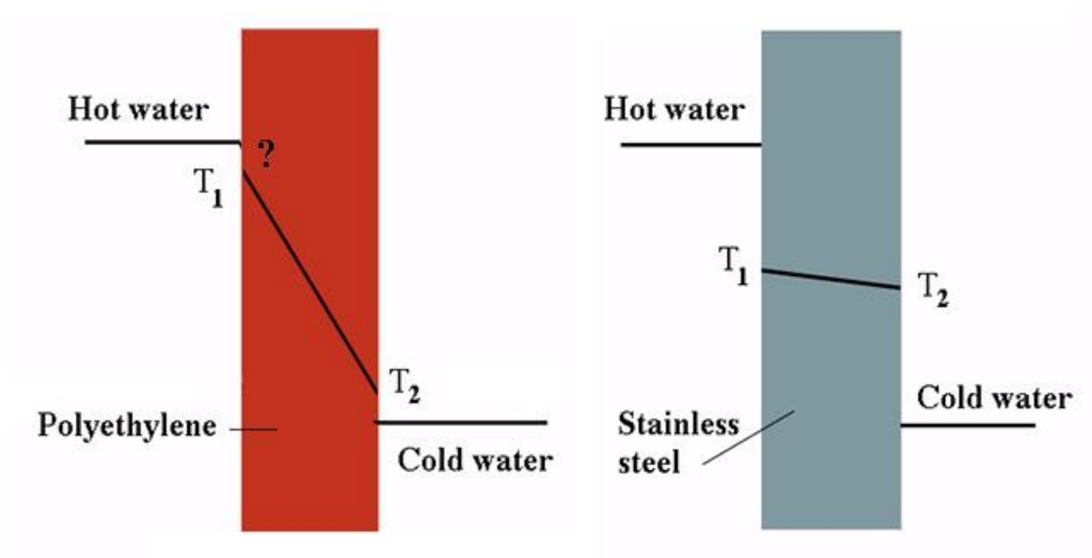


Fig 2 – schematic diagram showing very approximate inferred temperature profiles across the plastic and stainless steel walls.

The temperature gradients in walls is not the same, but there is another effect that we now need to consider.

Convective coupling

Heat is carried to and away from the wall in the water by convection. Turbulent mixing assisted by stirring maintains uniform temperatures away from the walls. Very close to the walls the water flow will be slower and more laminar in nature. Actually on the walls water will be stationary. As a first attempt to model the situation we will consider there to be thin layers of water on the walls of thickness Δx for which water flow is slow, laminar, and parallel to the wall. Heat is transported through thess boundary layers by conduction, governed by the same relationship that governs heat transfer in solids, namely …

$$\Delta Q/\Delta t = k_w A\ (\Delta T/\Delta x)$$

… where k_w is the conductivity of water, A is the area of the wall and $\Delta T/\Delta x$ is the unknown thermal gradient across the boundary layer.

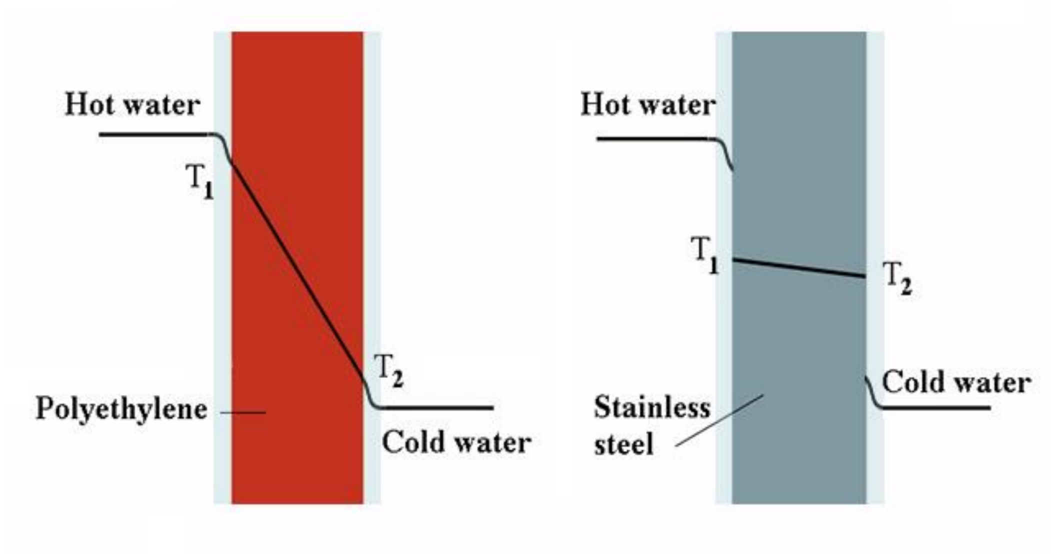


Fig 3 – schematic diagram showing suggested boundary layers in the water and very approximate inferred temperature profiles in the water close to the walls.

The thickness of the supposed boundary layer will be a function of the roughness of the wall, the viscosity of the water, and the speed of turbulent convection currents away from the wall. Possible values are open for discussion and investigation.

A project

As a first step towards an investigation water could be replaced by more viscous fluids (honey or heavy oil) inside, outside and/or both inside and outside. Water could be replaced by agar gel and the temperature gradients in the gel could be found with fine thermocouples. There are other approaches. The details are left for an open-ended project.