Demonstration: thermal expansion of water

Introduction

A glass cup is filled “to the brim” with water and heated to close to 100°C in a microwave oven. The cup is removed. Boiling water is added with a syringe to bring the water level above the rim as shown on the left. The cup and water are allowed to cool to room temperature (20°C). The water level falls. Adding 10 ml of water (2 tsp) on the right restores the water to the level on the left.

Fig 1 – The cup is full to the brim at 100°C (left) and not so full at 20°C (right).

What approximate error might be introduced by evaporation as the water in the cup cools over 20 minutes? If this loss is significant, how could it be reduced?

To restore the water level on the left about 10 ml (two teaspoons) of water must be added to the cup on the right. For more accuracy the hot water level could be adjusted so that a fine line just below the rim of the cup coincided with the bottom of the meniscus, and a cover could be put in place to reduce any evaporation during the cooing process.

Comment

This may be the first time the reader has noticed this effect. The expansion/contraction is a small percentage, almost never leads to overflow, and is seldom noticed. So the effect may be more clearly seen in figure 1 the mages have been inverted and sharpened.

The equations that describe expansion

The coefficient of volume expansion β_g is almost independent of temperature. When a glass cup is heated by ΔT from T_o to T …

$$V = V_o(1+\beta_g\,\Delta T) \qquad \dots\ [1]$$

… where V_o is the original volume at T_o

The coefficient of volume expansion $\beta_w(T)$ is a function of temperature. When water is heated by ΔT from T_o to T …

$$V = V_o(1 + \beta_w(T)\,\Delta T)$$

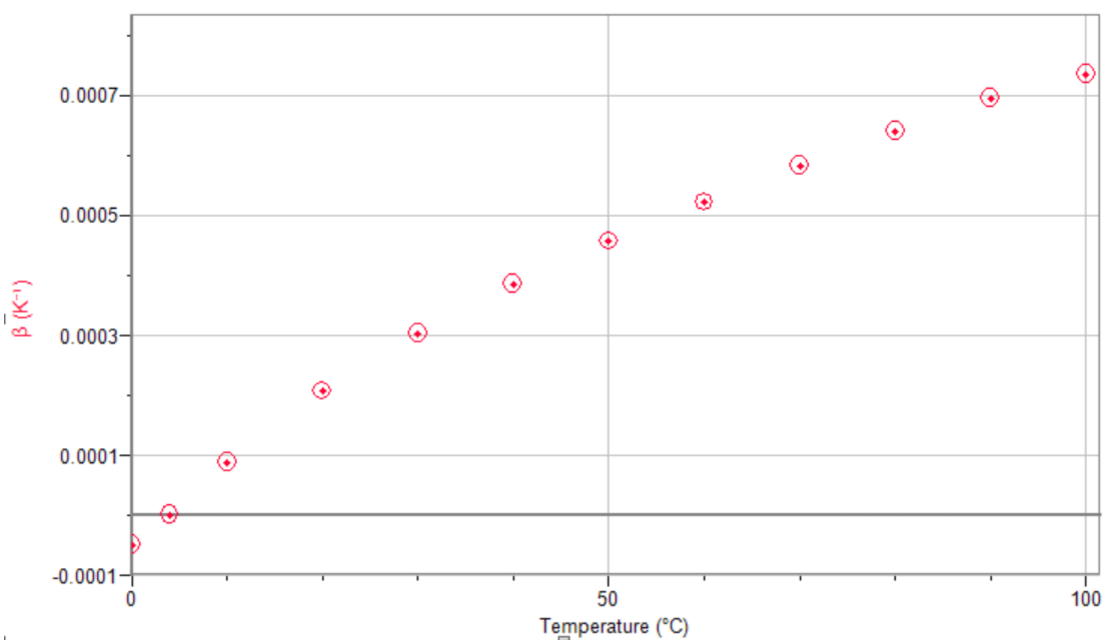
Rearranging gives …

$$\Delta V/V_o = \beta_w(T)\,\Delta T \qquad \dots\ [2]$$

For water, $\beta(T)$ is zero at 4°C and $7\text{x}10^{-4}\,K^{-1}$ at 100°C. Inserting a published graph into Logger pro allows data to be entered point by point, or data may be entered from a table of values.

See http://www.engineeringtoolbox.com/volumetric-temperature-expansion-d_315.html

The coefficient of thermal expansion for water



Graph 1 - Data from engineeringtoolbox.com entered point by point in Logger Pro.

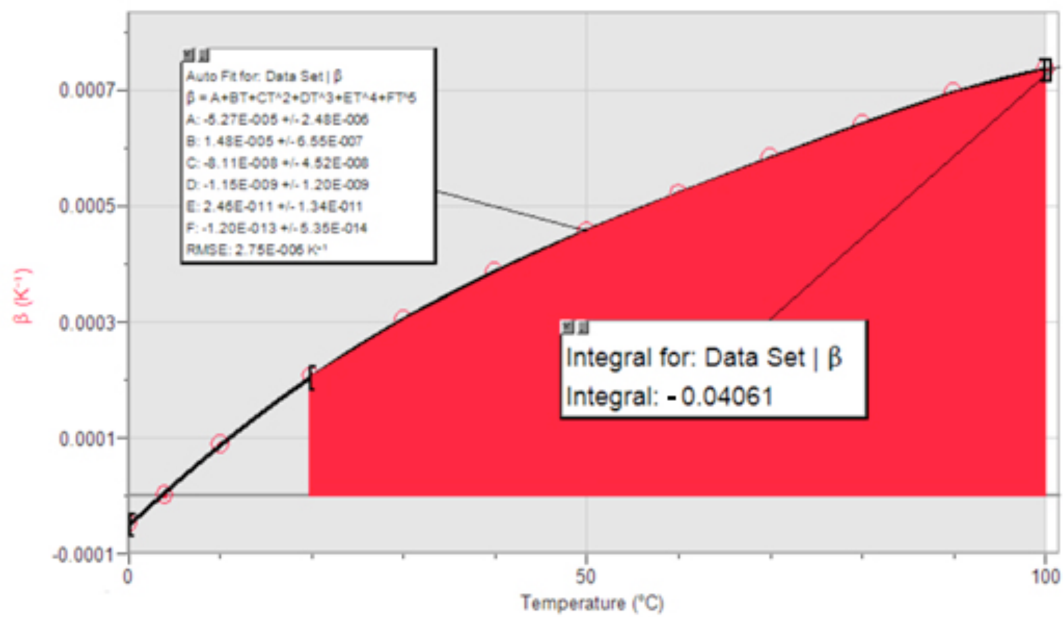
Calculations

Integrating both sides of equation 2, between V and $V+\Delta V$ on the left and T_1 and T_2 on the right, gives …

$$\ln\left(\frac{V+\Delta V}{V}\right) = \int_{T_1}^{T_2} \beta(T)\,dT$$

The log of a number is the power to which the base must be raised to get that number. The expression can be written as …

$$\frac{\Delta V}{V} = \exp\left(\int_{T_1}^{T_2} \beta(T)\,dT\right) - 1 \qquad \ldots [3]$$



Graph 2 - A polynomial fit to data from engineeringtoolbox.com.
The definite integral from 100-20°C is - 0.0406.

The volume reduction ΔV of 270 ml of water from 100-20°C is, from equation 3 …

$$270\,(2.71^{-0.0406} - 1) = 10.7 \text{ ml}$$

The volume reduction of the cup from 100-20°C ($\beta_g = 27\text{x}10^{-6}$) is, from equation 1 …

$$270\,(27\text{x}10^{-6})\,80 = 0.59 \text{ ml}$$

We see that 10.1 ml will restore the water level in the glass cup at 20°C.

Note: *Giancoli* (*Sixth Edition: New International Edition 2014*) gives 4.0 ml as the answer to a similar question, calculated incorrectly using β for water at 20°C ($2.1\text{x}10^{-4}$ K^{-1}).

Questions

1 Demonstrations are often more effective if done without the complications of complex plotting and measuring devices that can be distracting and take attention away from the essential point.

Which of the two things below (syringe and measuring spoons) do you suggest for adding a known volume of water to the cool cup and *explain why you make that choice.* One tsp (tea spoon) is 5 ml.

2 Use equation 3 to find the percentage volume reduction of a glass cup as a function of temperature.

3 Briefly outline why a water-in-glass thermometer is not normally useful.

4* With Logger Pro (or otherwise) calibrate demonstration thermometers: water/glass from 20 - 100°C and a water/PET-bottle from 0 - 70°C.

(Find a thermal expansion coefficient for **Polyethylene terephthalate**.)

Why is the temperature range for this 'thermometer' restricted to 0 - 70°C?

5* Numerical integration has been used (Graph 2) to find the definite integral.

Find the integral analytically by integrating the polynomial function, and approximately using $\beta_w(60^oC)$.