Demonstration: viscosity from first principles

Introduction

A friction force is very often independent of velocity. The relationship is not exact but is a good approximation and widely used: see *Skid Distance* in this index. A similar situation is shown in figure 1. The top plate that carries an ultrasound reflector is pulled by hand with the force probe and the velocity is recorded with the motion detector. Care is taken to align the force probe and to pull the plate slowly to maintain a slow shearing deformation of peanut butter between the plates.

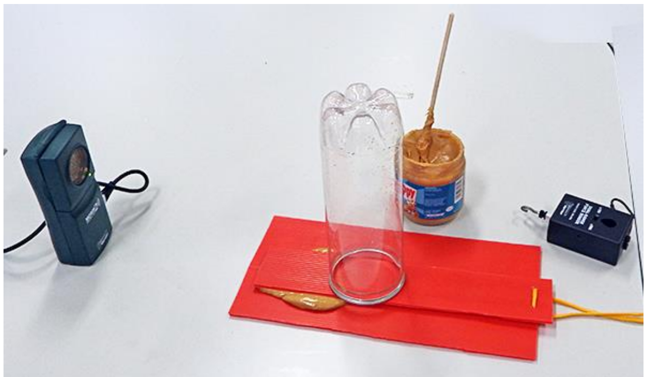

Fig 1 - Peanut butter, probes and a movable plate.

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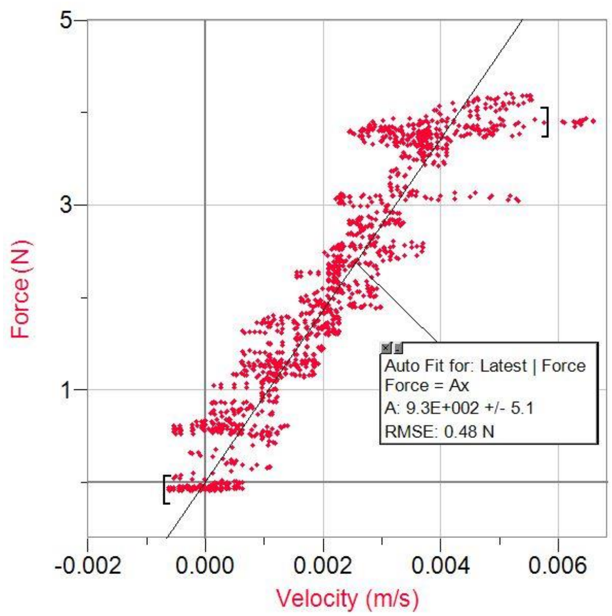
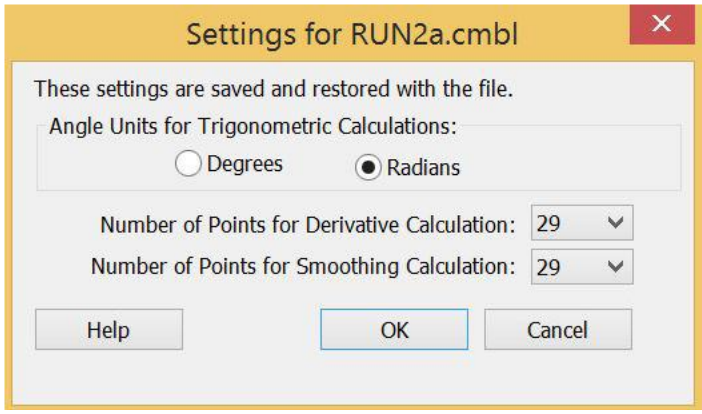


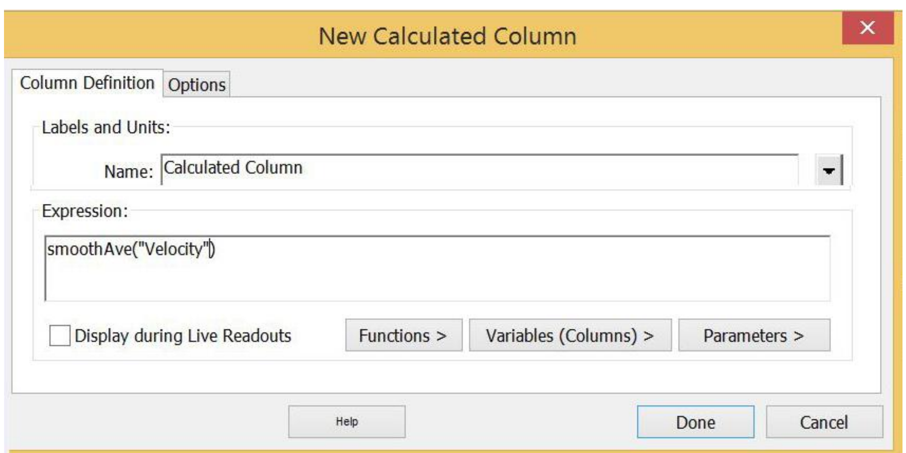
Graph 1 - The force required to move the plate is not constant but is proportional to velocity

Data points on Graph 1 are scattered because the motion detector is recording position data with small, but on this scale, significant errors. A more normal plot can be obtained with averaging and extending the number pf points used to calculate the derivative (*dx/dt*). Go to *Settings* in the *File* index in Logger pro and alter the number of points from 7 in each case to a higher number. Set the data rate to 50 points per second.



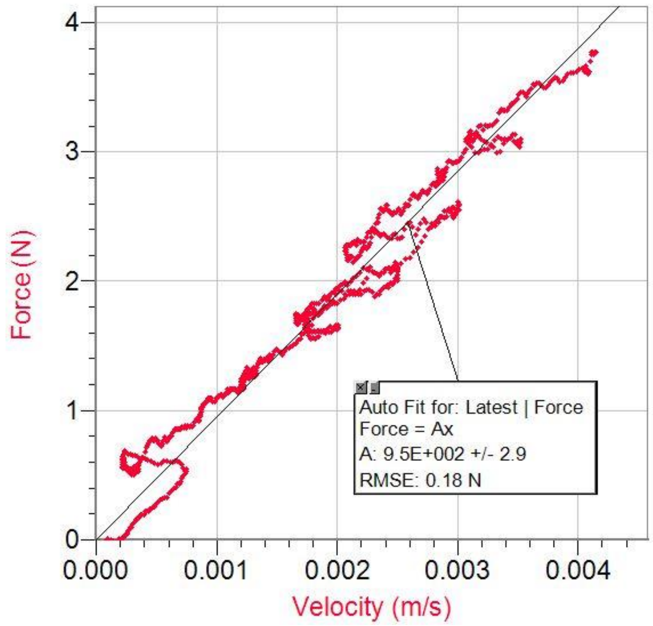
. **Fig 2 -** The settings window (part).

Averaging is done in a *New Calculated Column* by selecting *smoothAve* from the Functions list and entering the variable required from the Variables list.



. **Fig 2 -** The smoothing function window (part).

Note: *When demonstrating the effect for yourself in lab-time or for others in class alter the settings and save that Logger Pro file. Use that file to collect data displayed in real time with smoothing. Note that averaging cannot be done with digital data from a motion detector and smoothing after data collection in a calculated column is required.*



. **Graph 2 -** Force and velocity data with smoothing.

Note: For the linear relationship in Graph 2 to hold the peanut butter must adhere to the surfaces and move with them. The linear relationship breaks down at higher velocities when the peanut butter breaks away from the plate.

The coefficient of viscosity

Experimenting further with the apparatus in figure 1 shows that if the velocity v and layer thickness Δx are kept constant the drag force F required to move the plate at v is proportional to the area in contact A. If the velocity and area are kept constant the drag force required is inversely proportional to the layer thickness Δx. These results are expected and are confirmed by measurements.

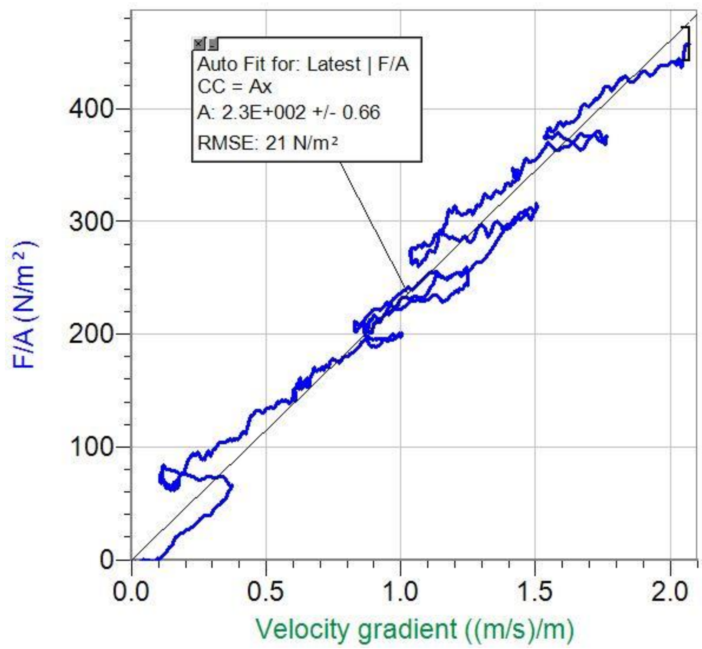
$$F = \eta \mathrm{A} (v/\Delta x)$$

The ratio F/A is defined as the *shearing stress*. The ratio $v/\Delta x$ is defined as the *velocity gradient*. With these definitions the viscosity eta is reduced to the ratio of just two quantities (shearing stress over velocity gradient).

$$\eta = (F/\mathrm{A})/(v/\Delta x)$$

Students sometimes fail to understand the definition of η presented in this way at a first reading, in part, there are four variables on the right hand side.

In this case the area of peanut butter in contact with the upper plate was approximately 5.5x15 cm. The thickness of the layer Δx was approximately 2 mm. Plotting force per unit area against the velocity gradient gives a straight line (Graph 3).



Graph 2 – Shearing stress plotted against the velocity gradient.

The slope of the line in Graph 3 is the viscosity of the peanut butter.

$$\eta = 230 \text{ Pa s}$$

Over the range of velocity gradients tested the value is constant.

Units

The SI unit (the pascal second) is defined as the poiseuille (symbol PI) but the equivalent cgs unit, the poise (symbol P) is more widely used when reporting viscosities.

$$1 \text{ Pa s} = 10 \text{ P}$$

Temperature dependence

Viscosity is a property of a fluid and is temperature dependent. In common speech the meaning is closest to “thickness” but there is no exact common equivalent in English. Viscosity decreases with temperature for liquids: honey and oil pour more easily when warm. Surprisingly the viscosity of a gas increases with temperature. The effect is partly explained by the kinetic theory of gasses and is the subject of a later post.