Work and Kinetic Energy

Introduction

The kinematic equations for linear motion for constant acceleration are well known. In particular …

$v^2 = u^2 + 2as$ … where the symbols have their usual meanings.

Substituting for a using $f = ma$, multiplying through by ½m and rearranging gives …

$$f.s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

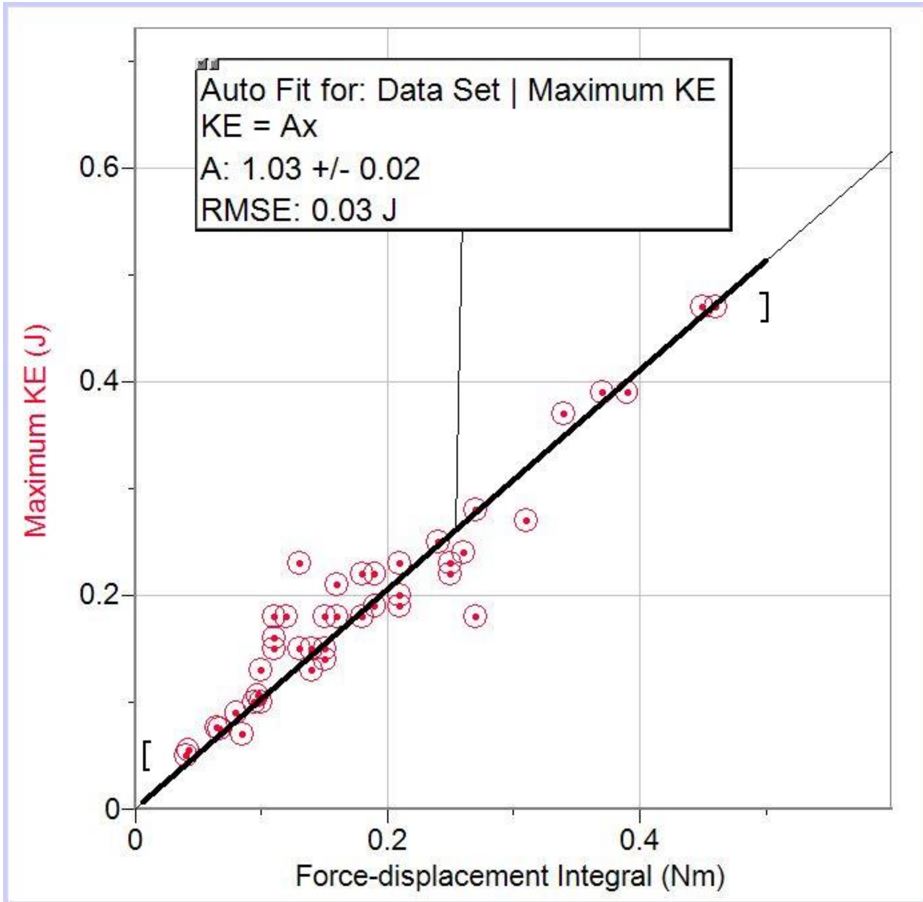
The right hand side of this equation is the change in what is now called *kinetic energy* and the left hand side is the area under a force displacement graph, for constant force, that is now called *work*. The equation suggests that when we have a constant accelerating force that the work done by the force is equal to the gain in kinetic energy. Elementary physics texts make this statement but historically that is far from obvious. The equation was widely known at least by 1700 but it was not until 1850 that the term kinetic energy was introduced after it became clear that heat was a form of energy (not a strange fluid).

In modern terms, when a force is not constant, $f.s$ becomes the force displacement integral where the displacement is in the direction of the force. The proposition may appear obvious to those with a physics education, but as always, theory must be verified by experiment.

M4 were asked to measure an accelerating force as a total force minus friction and to plot both force-position and velocity-time graphs. Since this was the student's first attempt and the force-position graphs were made with ten or less data points at 30 data points per second individual results were expected to have random errors of 10-20%. To reduce the effect of random errors many data points are plotted below.

Analysis

The calculated proportional line fit has a slope very close to 1.00 showing that the proposition: *work done equals kinetic energy gained* has been verified in this simple situation to within a few percent. The verification is very pleasing, given the simplicity of the apparatus, the inexperience of the personnel, and the errors inherent in the procedure (see below).



Graph 1 – kinetic energy gained versus the force-displacement integral for a non-constant accelerating force. Data from Kieo Auksarapak, Tinnakit Udsa, and associates.

Future work: to reduce errors

Data was obtained with total masses (cup plus weights) from 300-800 g with many points clustered in the lower range. In future the mass range could be increased to 1500 or 2000 g and the energy range could be increased by a factor of five.

The most important limitation was the small number of points on the force-position graphs. In future the acceleration could be done with a rubber band between the fore probe and the cup to extend the time for which the force was applied and to make the force-displacement graphs more regular.

The force probes were zeroed in the horizontal position but were not calibrated in this position. Errors of as much as 0.1 N may have been introduced and could be reduced by calibrating the force probes appropriately.

[Improved Data]