An Introduction to Wave Motion 1

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Angular motion

A radius vector of length *A* rotates about the origin with angular velocity ω in radians per second.

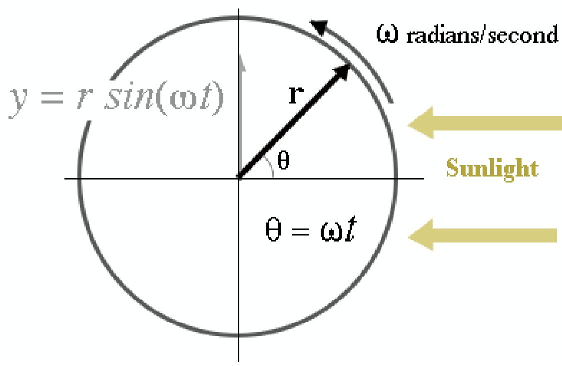


Fig 1 – the projection of a radius vector on the y axis is of length $r\sin\omega t$.

The length *y* is the *projection* of the radius vector on the vertical axis. (If the sun shines horizontally from the right, *y* is the length of the shadow of the radius vector on the *y* axis.)

$$y = A\,\sin(\omega t) \qquad \ldots$$

(*Construct a right angled triangle on the diagram and show this for yourself*.)

When the time passed is one period *T* we have $\omega T = 2\pi$.

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad \ldots 1$$

When displacement *y* equals $A\sin(\omega t)$ the velocity *v* is given by …

$$v = \frac{dy}{dt} = A\omega\cos(\omega t) \qquad \ldots 2$$

.. and the acceleration *a* is given by …

$$a = \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t) \qquad \ldots 3$$

Equations 1-3 are well known. They may be remembered and used without proof.

Simple harmonic motion

Suppose a heavy ball of mass *m* oscillates on a light hanging spring. Plotting the displacement-time graph gives data points that fit a sine function.

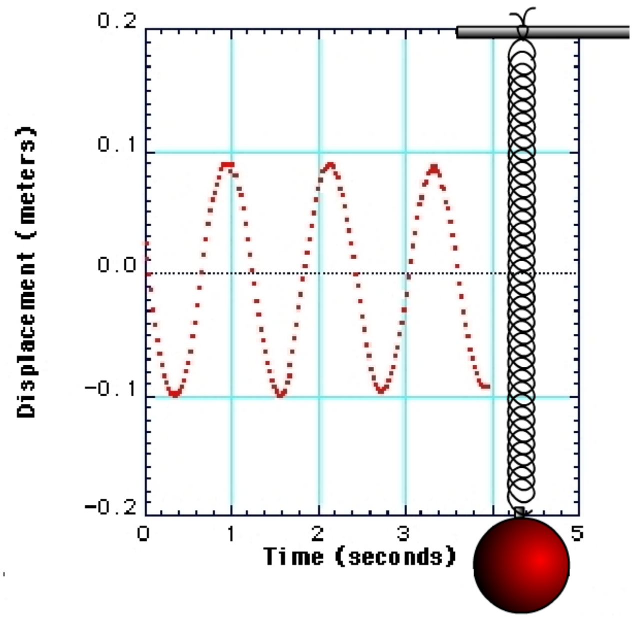


Fig 2 – The ball oscillates with simple harmonic motion [link].

Remembering that $f = ma$, and that for a spring $f = - ky$, gives the equation that describes the oscillation of the ball as …

$$m\frac{d^2y}{dt^2} = -ky$$

To show that this equation has a sine function solution we substitute for y from equation 1 and for the second derivative of y (the acceleration) from equation 3.

$$-mA\omega^2\sin(\omega t) = -kA\sin(\omega t)$$

Cancelling like terms on each side shows that the solution is correct when the constant ω equals the square root of the spring constant k over the mass m.

$$\omega = \sqrt{\frac{k}{m}}$$

Since the period T is $2\pi/\omega$, the period of the oscillation is given by …

$$T = 2\pi\sqrt{\frac{m}{k}}$$

… independent of amplitude.

Displacement velocity and acceleration for a ball on the light spring

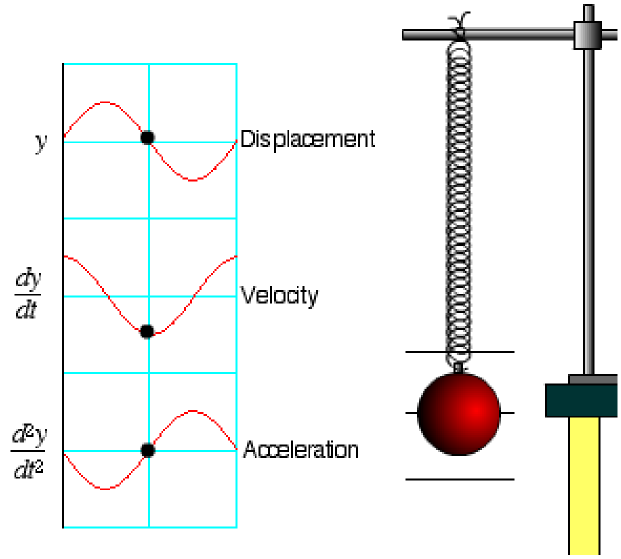


Fig 3 – displacement, velocity and acceleration for simple harmonic motion [link].

If the amplitude of the motion changes the period remains the same. Notice that at zero displacement force and acceleration are zero and velocity is a maximum, and … at maximum amplitude force and acceleration are maximum and velocity is zero.

Total energy

Method 1: The total energy of the simple harmonic motion is the PE stored in the spring plus the KE of the moving mass. The sum (PE + KE) will be the same at all times because there is no loss of energy to heat. When the ball is at amplitude *A*, the KE is zero and the PE is given by $^1/_2kA^2$. At all times …

$$PE + KE = {}^1/_2kA^2$$

Method 2: The energy of the motion at any given time is $({}^1/_2ky^2 + {}^1/_2mv^2)$. Substituting from equations 1 and 2 above gives …

$${}^1/_2\,k\,(A\sin\omega t)^2 + {}^1/_2m\,(A\omega\cos\omega t)^2 = {}^1/_2kA^2\,(\sin^2\omega t + \cos^2\omega t)$$

and … $PE + KE = {}^1/_2kA^2$ … as above.

Method 3:

Show that $PE + KE = {}^1/_2kA^2$ … by writing down the maximum KE.

Waves

The illustration below was captured from an animation (see the link in the cation). As the animation runs it is impossible to 'see' how the animation as made. The visual impression is of a fixed sine wave moving steadily to the right.

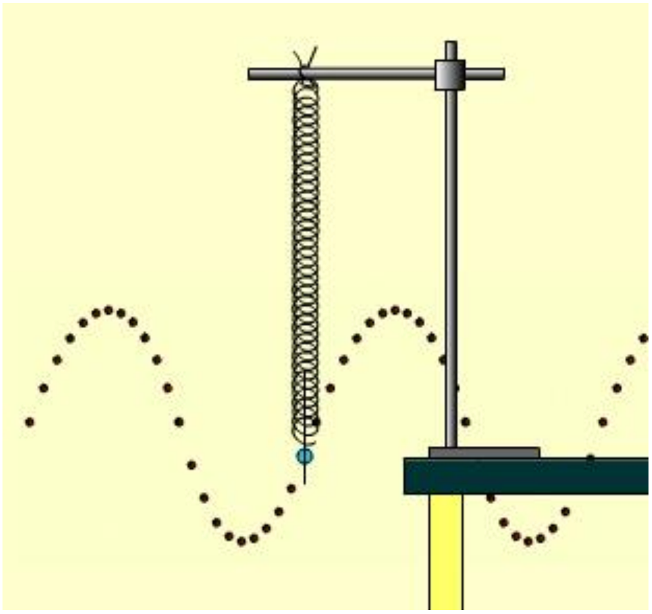


Fig 3 – a sine wave appears to move to the right as the blue dot oscillates vertically on the spring [link].

The animation was made this way …

1 A dot was set to oscillate with vertical SHM in layer 1 in Flash,

2 Layer 1 was copied and pasted to a new layer, displaced to the right and delayed by one unit of time, beginning a little later than the dot in layer 1.

3 Step 2 was repeated 57 times.

4 The oscillating spring diagram was pasted to layer 58 to illustrate the vertical simple harmonic motion of the blue dot.

As the animation runs the independent oscillating spots are seen as a travelling wave of fixed shape moving to the right.

The animation models a travelling wave as a set of harmonic oscillators. The model has many instructive features, but, as with all simple models there are limitations. As we shall see below the model is only strictly correct for small amplitude waves.

The wave equation … $v = f\lambda$

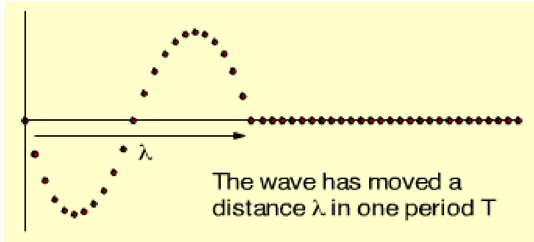


Fig 4 – a travelling sine wave moves to the right [link].

Velocity is distance over time. The wave moves to the right at a speed given by λ/T. Since the frequency f is one over the period T for *all waves* the velocity is given by …

$$v = f\lambda$$

Notes:

1 The wave equation is true for all waves. Sound waves, light waves, water waves, electromagnetic waves etc.

2 It is important to understand that frequency f is fixed by the *wave generator*.

Imagine that we have a straight line of lead balls connected by tensioned rubber bands. The ball on the left is suddenly forced upwards. As it begins to move tension rises in the first rubber band and the second ball begins to rise and so on. As time passes a wave travels to the right along the line of balls. If the balls have the same mass, are initially the same distance apart, and the rubber bands are all of the same strength (k), the wavelength and wave velocity will be the same everywhere. But, suppose the balls change in mass or separation along the line, or the rubber bands are stronger in different places. In each case the wavelength and wave velocity will change along the line of balls, but the ratio v/λ *is the oscillation frequency of the first ball and is fixed to the wave generator. The ratio* v/λ *will be the same everywhere along the line.*

3 It is important to understand that rigidity is impossible.

Rigidity is a simple mathematical concept, but an illusion in the real world. A marble behaves like a water balloon when it bounces on tiles. The tiles themselves are not rigid and neither is the earth as a whole. An earthquake in Japan makes waves in Bangkok.

A falling cup accelerates towards the floor as a whole, but when it hits the tiles, all that changes. If a local distortion exceeds a small limit it will break. If the distortions are small the cup will ring with standing waves that excite the surrounding air and we hear the result.

Questions

1 The equation represents a vertical simple harmonic motion.

$$y = A\ \sin(\omega t)$$

Which statement is not true?

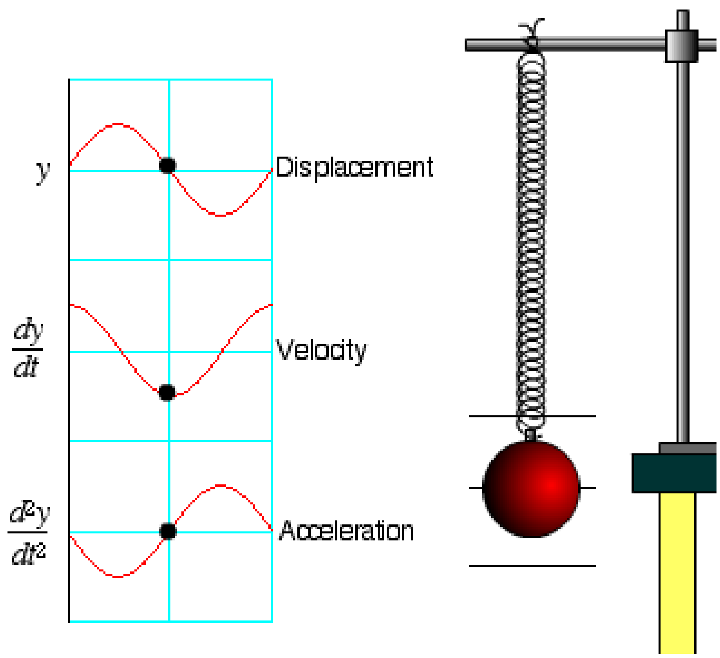
a The amplitude of the motion is *A*.

b The period of the motion is ω.

c The total energy of the motion is constant.

d The maximum velocity of the motion is *A*ω.

2 The figure shows a simple harmonic motion.



***Which statement is not true*?**

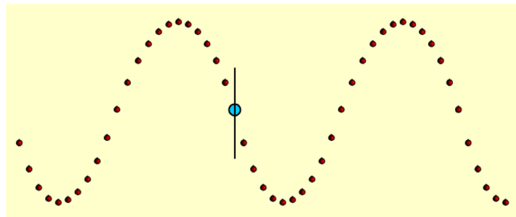
a The acceleration is zero as the red ball crosses the center line.

b The velocity is zero as the red ball crosses the center line.

c The force on the red ball is zero as it crosses the center line.

d The KE is a maximum when the red ball crosses the center line.

3 The figure has been captured from an animation that models a wave as a set of **equally spaced vertical harmonic oscillators**. (The dots can be imagined to be lead balls of the same mass equally spaced on a rubber bungee.)



***Which statement is tru*e?**

a The energy of each harmonic oscillation is the same.

b At this instant in time the blue dot is almost stationary.

c The wave is moving to the right.

d If the wave was on a long rubber rope the tension in the rubber would be the same everywhere.

4 Longitudinal waves are governed by the wave equation.

a A long heavy hanging sping is vibrated vertically at the top end with a frequency of 5000 Hz. Longitudinal waves propagate down the spring. Why is it not possible to define the velocity of these waves?

b A slinky is held at the top and hangs in a straight line. The top end is released. A compression pulse (shock front) travels down the spring from the top. The bottom coils remain stationary until the pulse reaches the bottom. Video clips are on the web. Many people are surprised and confused by this behavior and expect he spring to fall as a whole because:

a – they know that gravity affects each coil in the same way.

b – they forget that the hanging spring is not a rigid body.

c – the acceleration doe to gravity is the same everywhere.

d – they tried it in their imagination and that’s what happened.

5 a Find the frequency of atomic oscillations that generate light of wavelength 5×10^{-7} m.

b Describe one physical situation that gives rise to light of wavelengths in this wavelength region (5×10^{-7} m).