The speed of waves on a tensioned string

It can be shown that for small amplitude waves the speed of transverse waves on a tensioned string is given by the square root of the tension over the mass per unit length. The proof is omitted from first year university texts but may be found on the web.

$$v = \sqrt{\frac{T}{\mu}} \quad \text{… [1]}$$

The expression applies to small amplitude waves for which the medium is not distorted and the tension is the same at all points along the string. The approximation is expected to apply to standing waves on a violin string but may not be a good approximation for large amplitude standing waves on a rubber bungee.

Measurements

Remembering that standing waves are formed by the addition of reflected waves between to fixed points (closed boundaries), the velocity of the component waves is given simply by …

$$v = f\lambda \quad \text{… [2]}$$

Design and carry out measurements to verify relationship [1] for waves on a tensioned string for small amplitudes, a range of tensions, and for different mass per unit lengths for a material of your choice.

Note: The strings on a guitar could be used if high values of tension could be measured or a signal generator and vibrator could be used with tensioned steel wire or cord with a program like Audacity to measure frequency. Equipment is not available for these measurements.

Simpler methods are explored below.

Please read these accounts before designing your apparatus and procedures.

The speed of waves on a rubber bungee

Introduction

Waves can be created by hand on a long tensioned rubber band bungee. Standing waves set up between two fixed ends (closed boundaries) can be used to find the velocity of the component waves (incident and reflected) by measuring the frequency and measuring the wavelength.

A rubber bungee is shown in figure 1, stretched above a motion detector with a reflector (ping pong ball).

Fig 1 – tensioned rubber bungee and motion detector.

The rubber was set to oscillate with half a wavelength between the hand-held fixed ends with a small amplitude of less than ±2 cm. The frequency was found from the value of the constant ($B = 2\pi f$) in the sine function fit (figure 2).

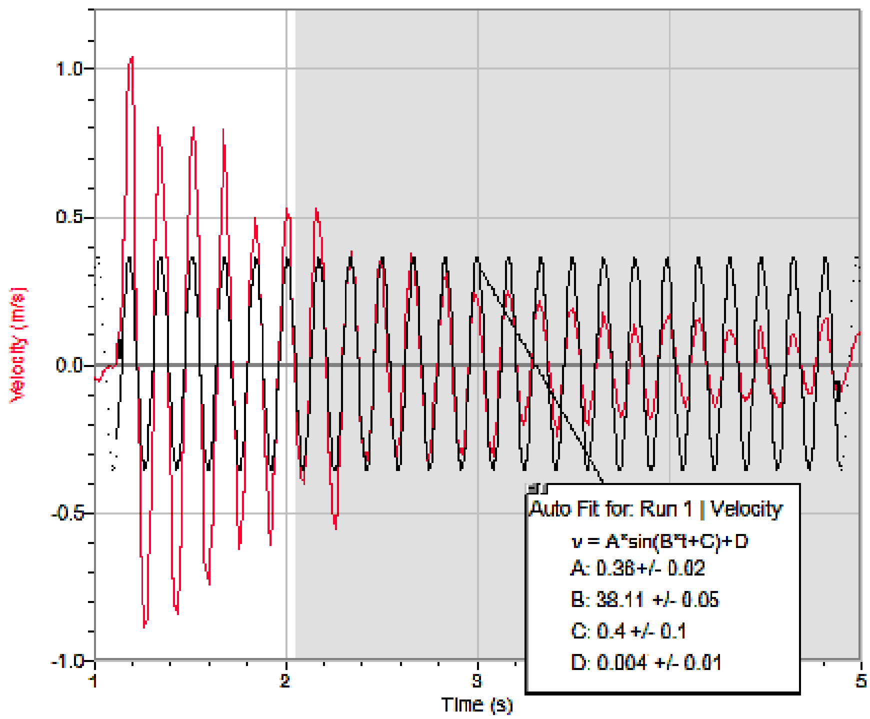


Fig 2 – oscillations and curve fit in Logger pro

Data Measurements of length, angular frequency, mass and tension made by three student groups was compiled in Logger pro. The tension in each case was measured with a spring balance and is the most uncertain of the four measured quantities.

Length (m)	Mass (kg)	ω (Rad/s)	Tension (N)	v (m/s)	fλ (m/s)
2.04	0.0119	34.6	3.2	23	22
1.49	0.0119	33.3	1.9	15	15
1.33	0.0119	24.2	1.4	12	10
1.06	0.0119	19.1	0.2	4	6
0.92	0.0107	26.2	0.4	5	7
1.08	0.0107	35.1	1.2	11	12
1.53	0.0107	41.5	2.9	20	20
2.3	0.0107	43.7	5.5	34	32
1.63	0.016	15.7	0.45	7	9
2.08	0.016	20.9	0.85	10	13
2.65	0.016	24.4	2.35	19	20

Fig 3 – student data.

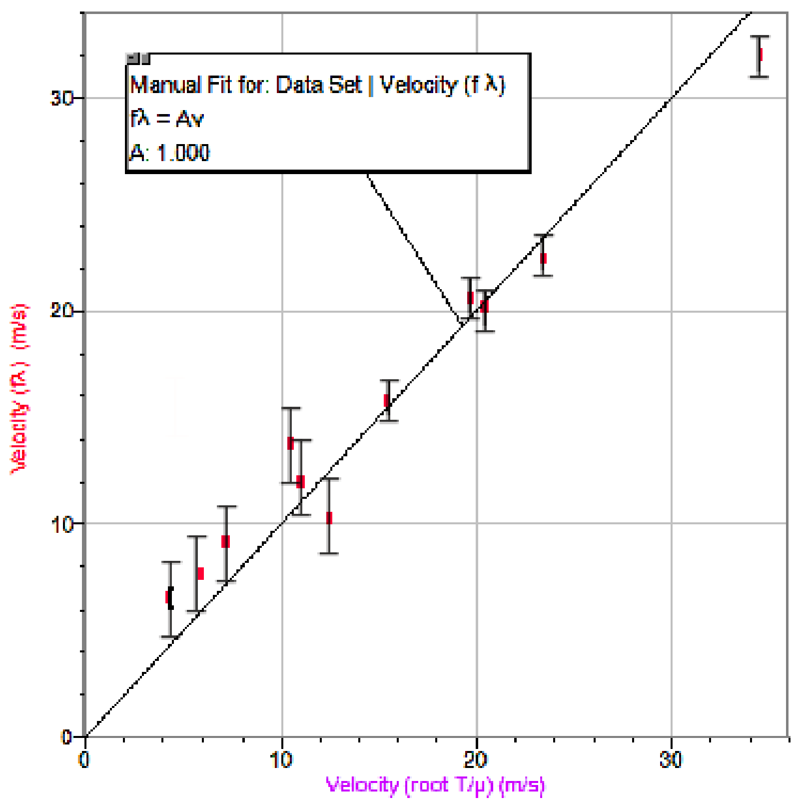
Analysis

The velocity of the component waves on the bungee was calculated using the velocity relationship in terms of the tension and mass per unit length …

$$v = \sqrt{\frac{T}{\mu}}$$

…. and was calculated with the wave equation using measured values of f and λ.

$$v = f\lambda$$



Graph 1 – velocities found in two different ways.

Discussion

Because the tension is most uncertain for small values, the low velocity measurements have large errors, but at higher velocities the errors are ±5%.The manual fit shows that within errors the two methods give the same velocities, showing that both relationships hold for small amplitudes. *Tensions could have been measured directly in real time with a force probe to reduce errors, especially at low tensions.*

Questions

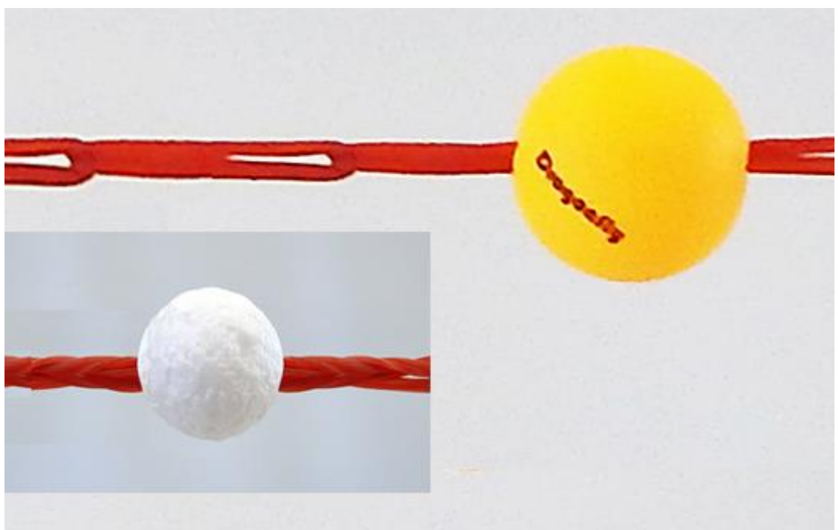
When finding the mass of the bungee with a balance one student included the mass of the pong pong ball.

Mass of the ball 2.36 g

Mass of the rubber 15.92 g

a *In your opinion was he correct to include the ball? Explain briefly why you think that.*

Our student was still unsure about what to do with the mass of the ball so he decided to look for a reflector with less mass. He replaced the ping pong ball with an expanded polystyrene ball with a mass of only 0.9 g.



He then attached the ends of the rubber firmly to a clamp stand and a fixed force probe to reduce errors in length measurement and he made a new bungee with four-band links to increase the mass per unit length. He measured the tensions with a force probe and as a final refinement he made the bungee close to two metres long (twice as long the original) to reduce the periods so that Logger Pro graphs had more data points per cycle at a maximum data rate of 39 points per second. In these ways he reduced errors by a factor of more than 10.

He was still unsure about what to do with the mass of the polystyrene ball.

b *What could he do to further reduce the effect of the mass of the ball?*