What *did* Carnot do?

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In 1824 Sadi Carnot published a remarkable little book in French: *Reflections on the Motive Power of Fire and on Machines Fitted to develop that Power.* A modern school physics text (sixth edition 2014) has it that Carnot defined the efficiency of an engine as $(Q_1\text{-}Q_2)/Q_1$ and showed that to be equal to $(T_1\text{-}T_2)/T_1$ for an ideal engine. [T_1 being the temperature of the hot side and T_2 being the temperature of the cold side]. *Given that Carnot did not have the ideal gas law, did not at the time understand that heat is a form of energy, and had no concept of entropy we might wonder how he did that.*

Selected details

From 1700 onwards engineers had tried to improve the efficiency of steam engines. By 1820 Watt in England had made some progress but they still thought they were wasting a lot of coal and had no idea why. Carnot was a well-educated French aristocrat, not a factory-floor man. While everyone else was confused with the details of clumsy dirty steam engines, he worked on paper with imaginary clean engines. That turned out to be a very clever idea indeed. Like most people at the time he described heat as a strange invisible fluid called caloric that flowed naturally from hot to cold bodies. He also believed (without really solid evidence) that engine efficiency didn't depend on the choice of working fluid, or on the *consumption* of caloric. In translation from the French

> … *"The production of motive power is due in steam engines not to actual consumption of caloric but to its fall from a warm body to a cold body."*

He did know that what he called the *fall of caloric* from a warm to a cold body is a one-way process. Heat (caloric) can never flow backwards from the cold body to the hot one by itself. To do that we have to use another engine. That is expensive. We want to get work out of an engine, not put it in.

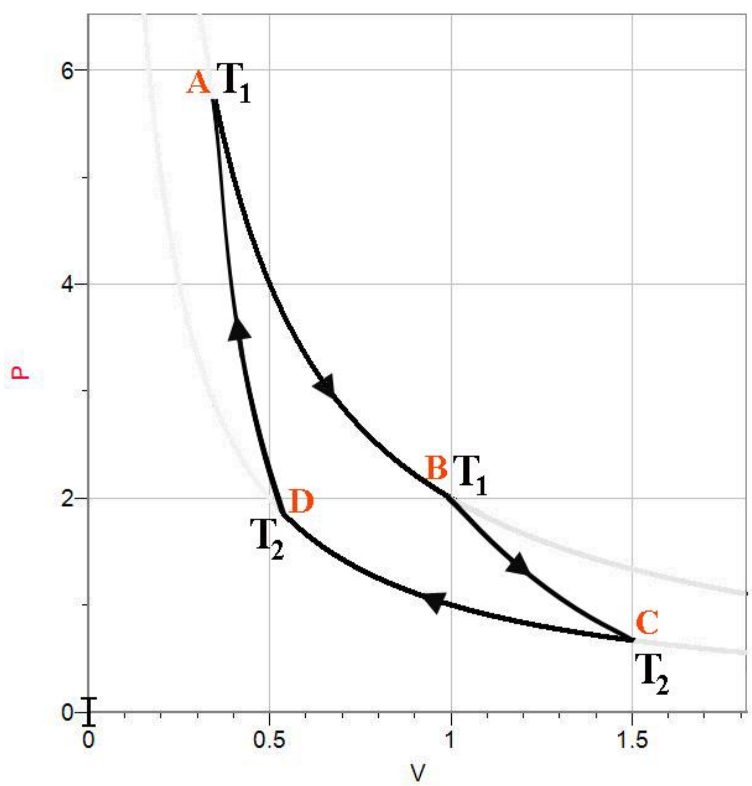
Carnot used an imaginary engine that had no natural transfer of heat (flow of caloric) from a warm place to a cold place. It sounds strange but on paper that is possible. In Carnot's terms there was no wasteful fall of caloric at any point on his closed cycle so his pencil-and-paper engine was reversible. Reversible has this exact meaning, which has nothing to do with drawing arrows the other way on a PV diagram.

A this point we are obliged to follow the practice of modern text books and draw his special cycle (the Carnot cycle) on a PV diagram but please note: just as Carnot did not derive the efficiency relationship in terms of T_1 and T_2, he did not do this. He described his cycle in clear explicit language: he understood exactly what he had done and why his engine was ideal in the sense that no other engine, real or imaginary, could be more efficient, but he did not draw the PV diagram. That came decades later from the pens of others.

Reversible paths on a PV diagram

There are two types of path on a PV diagram for which no (wasteful) transfer of heat takes place from a hot place to a cold place: an adiabatic process with no heat transfer at all, and an ideal isothermal process with heat transfer between bodies at the same temperature. We say *ideal* because, as Carnot knew, a slow isothermal process can be done with *almost no temperature difference* so on paper we can make the change as slowly as we like with ΔT as close to zero as we like. We will draw the Carnot Cycle on a PV diagram but remember, as we stressed above, Carnot himself described these changes clearly and carefully with no diagram.

Carnot's cycle: four reversible paths on a PV diagram for an ideal gas as working substance.



An ideal gas is confined to a cylinder fitted with a freely moving piston.

Slow isothermal expansion from A to B at temperature T_1 follows a hyperbola because PV is constant. Sudden adiabatic expansion from B to C lowers the temperature to T_2. Slow isothermal compression to D follows the lower hyperbola at T_2, and finally adiabatic compression returns the gas to its original state A at temperature T_1. The work done per cycle is the area enclosed by the loop. Carnot himself could not have known that, because he had not drawn the diagram.

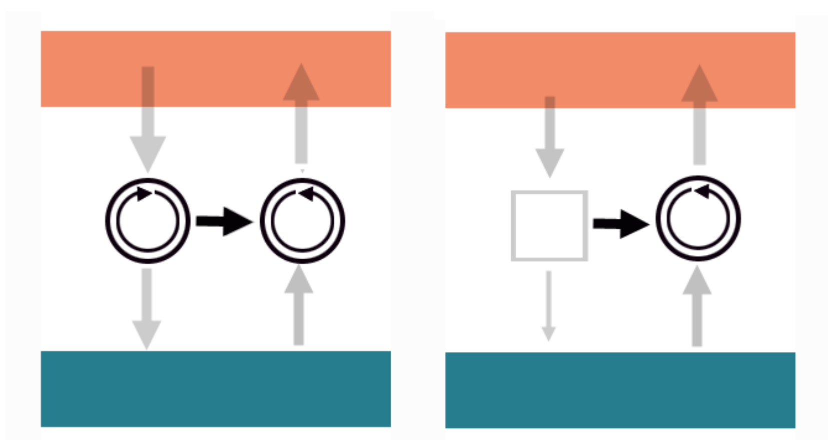
A reversible engine has maximum possible efficiency

Carnot's proof that any reversible engine has maximum possible efficiency showed very clear thinking. We shall put the proof in modern form without mentioning *the fall of caloric*, but we will use only words like he did. Read on very carefully and use your imagination.

A reversible engine takes heat Q from a hot reservoir, does work W, and exhausts heat (less than Q) to a cold reservoir. Work W from this engine drives a second engine in reverse and returns the same heat to the hot reservoir. The process is a waste of time (does no work and transfers not heat) but no physical laws are violated.

Now: suppose a more efficient engine takes less heat from the hot reservoir and produces the same work W. The work W is enough to drive our original engine in reverse to transfer the same heat as before to the hot reservoir. That would result in unaided heat flow from a cold place to a hot place which is impossible, so we see that no engine can be more efficient than a reversible engine, and that all reversible engines must have the same efficiency.

How do we know all this? We have his 1824 book. The book in English translation is very clear: but I find diagrams easier to remember. Carnot didn't draw diagrams so we can be creative and make up our own. I draw a reversible engine as a circle, and a hypothetical more efficient engine as a square. Engine diagrams are put between a hot place and a cold place and heat and work inputs and outputs are drawn as arrows.

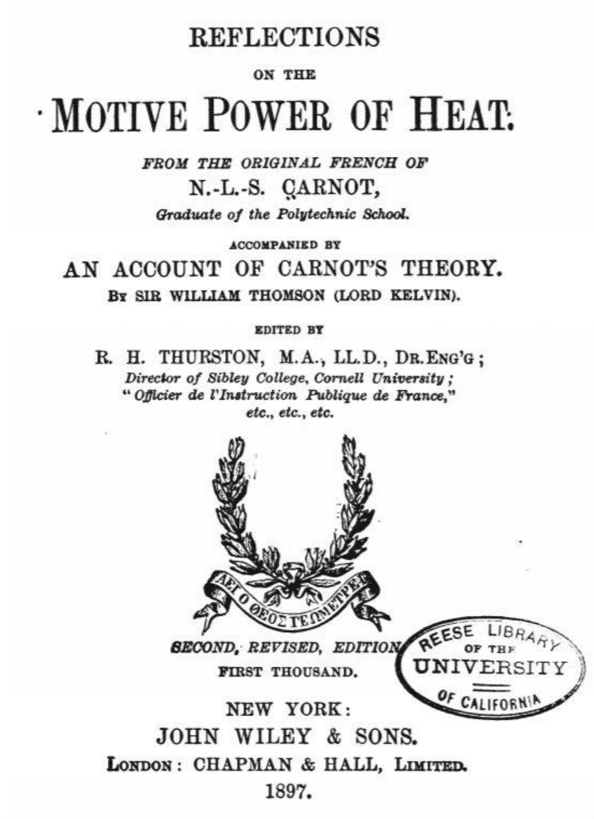


On the left the reversible engine takes in heat Q_1, converts some to work W and outputs Q_2. That work drives an identical engine that takes in Q_2 and outputs Q_1.

On the right my better engine takes in less heat than Q_1, does work W, runs a Carnot engine in reverse that outputs Q_1 to the hot reservoir, more than is being taken. Clever, but that can't happen. My better engine is a scam.

Appendix

The first page of an 1897 English translation of Carnot's book.



REFLECTIONS

ON THE

MOTIVE POWER OF HEAT.

FROM THE ORIGINAL FRENCH OF

N.-L.-S. CARNOT,

Graduate of the Polytechnic School.

ACCOMPANIED BY

AN ACCOUNT OF CARNOT'S THEORY.

BY SIR WILLIAM THOMSON (LORD KELVIN).

EDITED BY

R. H. THURSTON, M.A., LL.D., DR.ENG'G;

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etc., etc., etc.

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1897.

https://www3.nd.edu/~powers/ame.20231/carnot1897.pdf

To answer the question posed in the title of this reading … Carnot showed by careful argument that all ideal reversible cycles (including his example) have the same maximum possible efficiency that depends only on the difference between the hot and cold temperatures T_1 and T_2, but it was not until after Carnot's death in 1832 from cholera, that Clapeyron first used a PV diagram and defined the efficiency of an ideal reversible engine as $(Q_1 - Q_2)/Q_1$. It was not until 1854 that Clausius understood that for a Carnot Cycle $Q_1/T_1 - Q_2/T_2 = 0$ and the temperature relationship follows. The ratio Q/T is now defined as entropy … and that is another story.

For a brief history of thermodynamics try this account by John Murrell, on the web at … https://www.sussex.ac.uk/webteam/gateway/file.php?name=a-thermodynamics-history.pdf&site=35

[Language Questions]